Synthesis of Controllers for Modal Shaping in Linear Parameter-Varying Systems via the Implicit Model Following Formulation

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Abstract

The control synthesis problem involving Implicit Model Following (IMF) is considered in the context of Linear Parameter-Varying (LPV) and H_2/H_∞ theories. The well-known quadratic or nominal H_2 IMF problem is first extended to encompass LPV system models with a Linear Fractional Transformation (LFT) structure. This problem is then embedded in the framework of LPV theory. Conditions for dealing with additional mixed H_2/H_∞ criteria are discussed. The solvability conditions are provided with little conservatism by a previous multi-channel LFT/LPV result in discrete time. Finally, an illustrative example is used to validate this new formulation. Also, we demonstrate through this example that the IMF formulation is an effective technique to achieve a desired transient behavior for LPV systems.

1 Introduction

While most standard methods for robust control design of Linear Time-Invariant (LTI) systems focus on frequency domain specifications, in a number of applications many performance specifications are explicitly stated in time domain in terms of qualities of transient responses and internal state decoupling. It is well-known from classical control theory that the main properties of the time responses can be reflected in the frequency domain. Therefore, the performance objectives are often taken into account by choosing an appropriate synthesis structure and tuning frequency weighting functions, filters and/or dynamic scalings. Hence, the application of robust control design methods can lead to a large amount of trial-and-error before obtaining satisfactory conventional specifications in terms of time-domain properties.

Some robust synthesis methodologies, as those based on H_∞ model matching schemes and on robust pole placement approaches, handle time-domain specifications in a more explicit way. See, for instance, the references [11, 10, 6] and [3]. However, extra difficulties appear when non-stationary or nonlinear systems are considered, since they cannot be appropriately represented in the frequency domain. The pole notion no longer holds for these systems and some required transient properties are met only for slowly varying conditions. Moreover, because of excessive conservatism, these techniques are often restrictive in the multi-objective control and Linear Matrix Inequalities (LMI) [2, 5] contexts. Another drawback of the H_∞ model matching methods is that they generally produce high order controllers.

In reference [8], the authors present an alternative approach to deal with the control problem involving assignment of closed-loop modal shapes. The LTI IMF results of [7] are extended to the dynamic feedback case and reformulated in the H_2 context. In this method, time domain specifications are readily reflected in a quadratic criterion that penalizes the error between a desired dynamic behavior and that of the closed-loop system.

The purpose of this paper is to study the problem of achieving precise and robust time-domain specifications on specific states of non-stationary LPV systems using IMF and a multi-channel LFT/LPV control method.

2 Problem Statement

Consider a continuous-time LPV plant with LFT structure

$$\begin{aligned} \begin{bmatrix} \dot{x}(t) \\ z_\Delta(t) \\ z(t) \\ y(t) \end{bmatrix} &= \begin{bmatrix} A & B_\Delta & B_1 & B_2 \\ C_\Delta & D_{\Delta\Delta} & D_{\Delta 1} & D_{\Delta 2} \\ C_1 & D_{\Delta 1} & D_{11} & D_{12} \\ C_2 & D_{2\Delta} & D_{21} & D_{22} \end{bmatrix} \begin{bmatrix} x(t) \\ w_\Delta(t) \\ w(t) \\ u(t) \end{bmatrix} \\ w_\Delta(t) &= \Delta(t)\, z_\Delta(t), \end{aligned} \quad (1)$$

where $A \in \mathbf{R}^{n\times n}$, $\Delta(t) \in \mathbf{R}^{N\times N}$, $D_{12} \in \mathbf{R}^{p_1\times m_2}$ and $D_{21} \in \mathbf{R}^{p_2\times m_1}$ define the problem dimension. The notation for signals is standard: x for the state vector, w for exogenous inputs, z for controlled or performance variables, u for the control signal, and y for the measurement signal. $\Delta(t)$ is a time-varying matrix-valued parameter evolving in a polytopic set $\mathcal{P}_{\mathbf{\Delta}}$, with

$$\mathcal{P}_{\mathbf{\Delta}} := co\{\Delta_1, \ldots, \Delta_i, \ldots, \Delta_L\} \ni 0\,, \qquad (2)$$

where *co* stands for the convex hull and the Δ_i's denote the vertices of $\mathcal{P}_{\mathbf{\Delta}}$. That is,

$$\Delta := \sum_{i=1}^{L} \alpha_i \Delta_i, \qquad \sum_{i=1}^{L} \alpha_i = 1, \qquad (3)$$

where $\alpha_i \geq 0$ are the polytopic coordinates of Δ. Polytopic coordinates are computed in real time as functions of the scheduling variables and can be exploited by the controller. According to our definitions, the pairs (w, z) and (w_Δ, z_Δ) define the performance and the gain-scheduling channels, respectively. For simplicity of presentation, we assume that $D_{22} = 0$ which incurs no loss of generality.

For the LPV plant (1) the gain-scheduling control problem consists in seeking an LPV controller with LFT structure

$$\begin{bmatrix} \dot{x}_K(t) \\ u(t) \\ z_K(t) \end{bmatrix} = \begin{bmatrix} A_K & B_{K1} & B_{K\Delta} \\ C_{K1} & D_{K11} & D_{K1\Delta} \\ C_{K\Delta} & D_{K\Delta 1} & D_{K\Delta\Delta} \end{bmatrix} \begin{bmatrix} x_K(t) \\ y(t) \\ w_K(t) \end{bmatrix}, \qquad (4)$$
$$w_K(t) = \Delta_K(t) z_K(t),$$

where $A_K \in \mathbf{R}^{n\times n}$ and $\Delta_K \in \mathbf{R}^{N\times N}$, such that H_2 and/or H_∞ specifications are achieved for a family of channels (w_j, z_j), $j = 1, 2, \cdots$, where the w_j's and z_j's are sub-vectors of w and z, respectively (Figure 1). In other words, bounds ν_j on the variance of the outputs z_j and/or bounds γ_j on the L_2-induced gain of the operator mapping w_j into z_j are guaranteed for all parameter trajectories $\Delta(t) \in \mathcal{P}_{\mathbf{\Delta}}$. The notation Δ_K is used for the controller gain-scheduling function which is a function of the parameter Δ, that is, $\Delta_K := \Delta_K(\Delta)$.

Remark 2.1 *For an H_2 performance index ν_j to be well defined in continuous time, the state-space data must be such that the closed-loop feedthrough term of the channel/specification j is zero. Without imposing restrictions to the controller, this is achieved with $D_{11j} = 0$ and either $D_{1\Delta j} = 0$ and $D_{12j} = 0$ or $D_{\Delta 1j} = 0$ and $D_{21j} = 0$.*

3 H_2−optimal IMF with LFT/LPV Structure and Output Feedback

In this section, we revisit the IMF formulation within the context of LPV systems and their associated control problem in a new statement of earlier results in quadratic IMF is given taking into account the class of LFT/LPV systems in (1). More specifically, the IMF problem is interpreted as an H_2 specification of the multi-channel LFT/LPV problem stated in the previous section. As a first phase, we must determine matrices

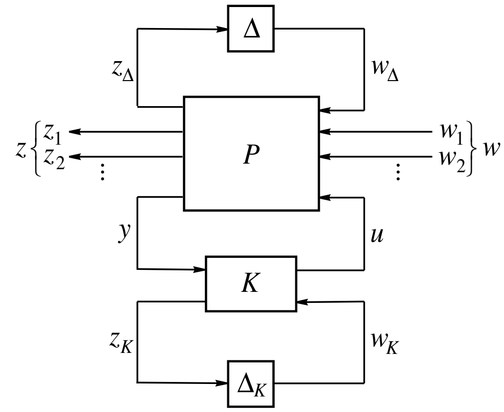

Figure 1: Mixed H_2/H_∞ multi-channel LPV interconnection

$$\begin{bmatrix} B_1 \\ D_{\Delta 1} \\ D_{11} \\ D_{21} \end{bmatrix}_{imf} \quad \text{and} \quad [\,C_1 \quad D_{1\Delta} \quad D_{11} \quad D_{12}\,]_{imf} \qquad (5)$$

that define a fictitious pair (w_{imf}, z_{imf}) and a corresponding H_2 performance channel for the system (1) which reflect the IMF problem.

Ignoring the performance channel for a moment and closing the Δ-loop, the system (1) is described as:

$$\begin{aligned} \dot{x} &= \left[A + B_\Delta \hat{\Delta} C_\Delta\right] x + \left[B_2 + B_\Delta \hat{\Delta} D_{\Delta 2}\right] u \\ y &= \left[C_2 + D_{2\Delta} \hat{\Delta} C_\Delta\right] x\,, \end{aligned} \qquad (6)$$

where

$$\hat{\Delta} = \Delta\left[I - D_{\Delta\Delta}\Delta(t)\right]^{-1}. \qquad (7)$$

Let $\xi(t) \in \mathbf{R}^q$ be an additional variable and $\mathcal{H} \in \mathbf{R}^{q\times n}$ a full rank matrix that selects some important modes from the state vector, namely

$$\xi(t) = \mathcal{H}x(t)\,. \qquad (8)$$

The IMF problem consists in finding a dynamic output feedback control law (4), $u = \mathcal{F}_l(K, \Delta_K)y$, where $\mathcal{F}_l$ is the notation for lower LFTs, for (6) such that the closed-loop dynamics of the controlled output ξ are as close as possible to those of the desired dynamics, given by

$$\dot{\eta}(t) = A_d \eta(t)\,, \quad \eta(t) \in \mathbf{R}^q, \qquad (9)$$

for all admissible parameter trajectories $\Delta(t) \in \mathcal{P}_{\mathbf{\Delta}}$. Note that A_d is usually selected to reflect time-domain specifications.

With the IMF paradigm, $\eta(t) = \xi(t)$, one can compute the error derivative from (6) and (9):

$$\begin{aligned} \dot{e} &:= \mathcal{H}\dot{x} - \dot{\eta} \\ &= \left[\mathcal{H}\left(A + B_\Delta \hat{\Delta} C_\Delta\right) - A_d \mathcal{H}\right] x \\ &\quad + \mathcal{H}\left(B_2 + B_\Delta \hat{\Delta} D_{\Delta 2}\right) u\,. \end{aligned} \qquad (10)$$

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The above problem can then be recast as minimizing a quadratic performance index of the form

$$J_{imf} := \int_0^\infty \left(\dot{e}^T R_0 \dot{e} + u^T R_1 u\right) dt\,, \qquad (11)$$

where an input weight R_1, a weighting, has been introduced for design flexibility. By substituting (10) into (11) this criterion becomes

$$J_{imf} = \int_0^\infty \left[x^T R_x x + 2x^T R_{xu} u + u^T R_u u\right] dt, \quad (12)$$

where the weighting matrices R_x, R_{xu} and R_u are given on the top of the next page. Thus, analogously to previous results in the LTI context, the H_2-optimal IMF with LFT/LPV structure results in a minimization of a standard quadratic criterion with a cross-weighted term $x^T R_{xu} u$. This problem can then be restated as an LFT/LPV control problem consisting in minimizing an upper bound ν_{imf} on the H_2 performance index of the channel (w_{imf}, z_{imf}) of the system (1) defined by the matrices

$$\begin{bmatrix} B_1 \\ D_{\Delta 1} \\ D_{11} \\ D_{21} \end{bmatrix}_{imf} = \begin{bmatrix} I_n \\ 0_{N\times n} \\ 0_{(q+m_2)\times n} \\ 0_{p_2\times n} \end{bmatrix} \qquad (13)$$

and

$$[C_1 \quad D_{1\Delta} \quad D_{11} \quad D_{12}]_{imf} =$$

$$\left[\begin{array}{c|c|c|c} C_{11} & R_0^{1/2}\mathcal{H}B_\Delta & 0_{q\times n} & R_0^{1/2}\mathcal{H}B_2 \\ 0_{m_2\times n} & 0_{m_2\times N} & 0_{m_2\times n} & R_1^{1/2} \end{array}\right], \qquad (14)$$

where

$$C_{11} = R_0^{1/2}(\mathcal{H}A - A_d\mathcal{H})\,.$$

In other words, the LFT/LPV H_2 IMF problem seeks an LPV controller (4) that minimizes the worst-case (with respect to Δ) energy of the output z_{imf} in response to impulse inputs in w_{imf}. Notice that the conditions in Remark 2.1 are met and this problem is properly posed.

It is important to stress the fact that the derived IMF formulation is well-suited to the multi-channel/specification context allowing to incorporate a set of additional H_2 and/or H_∞ performance constraints. In [1] the solvability conditions of the mixed H_2/H_∞ multi-channel problem in discrete time are provided with little conservatism and the synthesis LMI characterizations up to the discrete-time LPV controller construction are shown explicitly. The paper gives a comprehensive description of the proposed methodology while its applicability to a realistic continuous-time LPV system is investigated in [9].

4 Multi-objective LFT/LPV Result

In this section, we summarize the LPV control synthesis procedure proposed in references [1, 9]. The general synthesis scheme is described below.

Algorithm 4.1 *Controller synthesis*

Step 1: *Given the continuous-time plant (1),*

$$\begin{array}{rcl} \dot{x}(t) & = & Ax(t) + B\vartheta(t) \\ \psi(t) & = & Cx(t) + D\vartheta(t), \end{array} \qquad (15)$$

with $\vartheta := [w_\Delta^T,\ w^T,\ u^T]^T$ *and* $\psi := [z_\Delta^T,\ z^T,\ y^T]^T$, *compute its corresponding discrete-time state description by applying the bilinear transformation*

$$\tilde{P} := \begin{bmatrix} \tilde{A} & \tilde{B} \\ \tilde{C} & \tilde{D} \end{bmatrix} = \mathcal{F}_u\left(\begin{bmatrix} A & B \\ C & D \end{bmatrix}, \begin{bmatrix} I & \sqrt{2}I \\ \sqrt{2}I & I \end{bmatrix}\right), \qquad (16)$$

where $\mathcal{F}_u$ *is the notation for upper LFTs.*

Step 2: *Define the set* $\boldsymbol{G_v}$ *of general non symmetric decision variables which are common to all specifications and channels and consist of general slack variables, transformed controller variables and scheduling function coefficients (a precise definition of the set* $\boldsymbol{G_v}$ *can be found in references [1] and [9]).*

Step 3: *For each* H_2*-channel, define the set* $\boldsymbol{H_{2v}}_j$ *of the following symmetric decision variables:* H_2 *Lyapunov variables,* H_2 *scaling variables and a performance variable* ν_j.

Step 4: *For each* H_∞*-channel, define the set* $\boldsymbol{H_{\infty v}}_j$ *of the following symmetric decision variables: an* H_∞ *Lyapunov variable,* H_∞ *scaling variables and a performance variable* γ_j.

Step 5: *For each channel/specification, construct the LMI constraint system derived in Appendix A of [1] and represented here by the simple notations below:*

- H_2 *performance:*

$$\mathcal{L}_{H_2}(\boldsymbol{G_v}, \boldsymbol{H_{2v}}_j, \{\Delta_i\}, \tilde{P}_j) < 0 \qquad (17)$$

- H_∞ *performance:*

$$\mathcal{L}_{H_\infty}(\boldsymbol{G_v}, \boldsymbol{H_{\infty v}}_j, \{\Delta_i\}, \tilde{P}_j) < 0 \qquad (18)$$

where $\tilde{P}_j$ *is the set of discrete-time state-space matrices (16) representing the LPV plant (1) with only the channel/specification* (w_j, z_j) *under consideration.*

Step 6: *(LMI optimization problem) - Basically, three kinds of problems can be formulated:*

$$\begin{aligned} R_x &= \left[\mathcal{H}\left(A+B_\Delta\hat{\Delta}C_\Delta\right)-A_d\mathcal{H}\right]^T R_0\left[\mathcal{H}\left(A+B_\Delta\hat{\Delta}C_\Delta\right)-A_d\mathcal{H}\right] \\ R_{xu} &= \left[\mathcal{H}\left(A+B_\Delta\hat{\Delta}C_\Delta\right)-A_d\mathcal{H}\right]^T R_0\left[\mathcal{H}\left(B_2+B_\Delta\hat{\Delta}D_{\Delta 2}\right)\right] \\ R_u &= \left[\mathcal{H}\left(B_2+B_\Delta\hat{\Delta}D_{\Delta 2}\right)\right]^T R_0\left[\mathcal{H}\left(B_2+B_\Delta\hat{\Delta}D_{\Delta 2}\right)\right]+R_1 \end{aligned}$$

- *(Single H_2 or H_∞ synthesis) Minimize a specific performance variable γ_j or ν_j subject to the LMI constraints (17) and (18), keeping the remaining performance variables below some adequate set of values ($\gamma_\ell < \overline{\gamma}_\ell$ or $\nu_\ell < \overline{\nu}_\ell$, $\ell = 1, 2, ..., \ell \neq j$), that is, setting them at some chosen constant values;*
- *(Weighted mixed H_2/H_∞ synthesis) Minimize a trade-off criterion of the form*

$$\sum_j (\alpha_j\gamma_j + \beta_j\nu_j),$$

under the LMI constraints (17) and (18), where the α_j (≥ 0) and β_j (≥ 0) are scalar weights, imposing or not some adequate set of upper-bound constraints $\gamma_j < \overline{\gamma}_j$ and/or $\nu_j < \overline{\nu}_j$;

- *(Feasibility problem) Compute a feasible solution to the LMI constraints (17) and (18), imposing or not upper-bound constraints $\gamma_j < \overline{\gamma}_j$ and/or $\nu_j < \overline{\nu}_j$.*

Step 7: *As described in [1], compute the discrete-time LPV controller data,*

$$\tilde{K} := \begin{bmatrix} \tilde{A}_K & \tilde{B}_{K1} & \tilde{B}_{K\Delta} \\ \tilde{C}_{K1} & \tilde{D}_{K11} & \tilde{D}_{K1\Delta} \\ \tilde{C}_{K\Delta} & \tilde{D}_{K\Delta 1} & \tilde{D}_{K\Delta\Delta} \end{bmatrix}, \qquad (19)$$

as functions of the decision variables obtained in Step 6. The controller gain-scheduling function is determined by

$$\Delta_K(\Delta) := \sum_{i=1}^{L} \alpha_i \Phi_i, \qquad (20)$$

where the Φ_i can be computed off line as functions of the decision variables.

Step 8: *Using the result in (19), recover the corresponding continuous-time LPV controller data (4),*

$$K := \begin{bmatrix} A_K & B_{K1} & B_{K\Delta} \\ C_{K1} & D_{K11} & D_{K1\Delta} \\ C_{K\Delta} & D_{K\Delta 1} & D_{K\Delta\Delta} \end{bmatrix},$$

by applying the inverse bilinear transformation

$$K = \mathcal{F}_u\left(\tilde{K}, \begin{bmatrix} -I & \sqrt{2}I \\ \sqrt{2}I & -I \end{bmatrix}\right).$$

The introduction of new linearizing transformation variables and of general matrix variables leads to a full LMI characterization of the LPV control problem allowing the use of multiple Lyapunov functions and scalings known to reduce conservatism. The idea of using general slack variables to get rid of the standard common Lyapunov and scaling terms, that generally impose strong limitations, have been presented earlier in [4] for LTI multi-objective synthesis.

5 Illustrative Example

The goal of this section is to validate the new LFT/LPV IMF formulation developed in Section 3 by running the Algorithm 4.1. Given our specific interest on the IMF objective, additional specifications or channels are not considered here.

Consider the translational second-order spring-mass-damper system described as:

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ \frac{-k(\Delta(t))}{m} & \frac{-f}{m} & \frac{1}{m} \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ u(t) \end{bmatrix}. \qquad (21)$$

The states x_1 and x_2 are, respectively, the displacement around the equilibrium position (system at rest) and the velocity of the mass m. The control signal u is a force applied to the mass m and the output y is the displacement measured from the equilibrium position, $y = x_1$. The viscous friction coefficient f is considered constant, while the stiffness factor k is time dependent and varies around a constant value k_0:

$$k = k_0 + \frac{\Delta(t)}{2} \ \ \text{Nm}^{-1}, \qquad \Delta(t) \in [-1, 1].$$

Defining $z_\Delta := x_1$, an exact and minimal LFT representation of the system (21) is readily found to be

$$\begin{aligned} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ z_\Delta \\ y \end{bmatrix} &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{-k_0}{m} & \frac{-f}{m} & \frac{-1}{2m} & \frac{1}{m} \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ w_\Delta \\ u \end{bmatrix}, \\ w_\Delta(t) &= \Delta(t) z_\Delta(t). \end{aligned} \qquad (22)$$

Assuming that $m = 1$ Kg, $f = 1.2$ Nsm^{-1}, and $k_0 = 1$ Nm^{-1}, and using the SIMULINK® model depicted in Figure 2, we obtain the open-loop responses to a unit step described in Figure 3. One can notice that the dynamic behavior of the system is very sensitive to parameter variations.

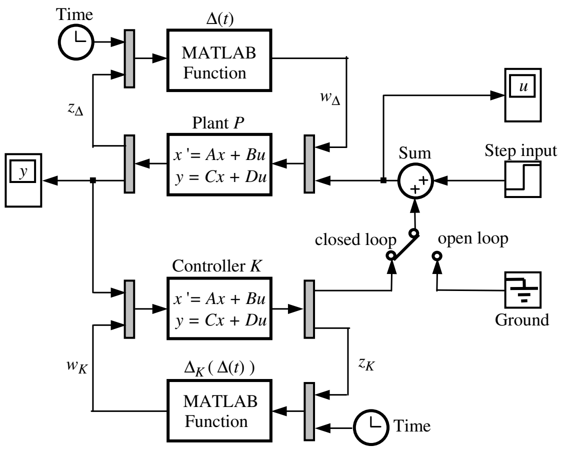

Figure 2: Simulation diagram

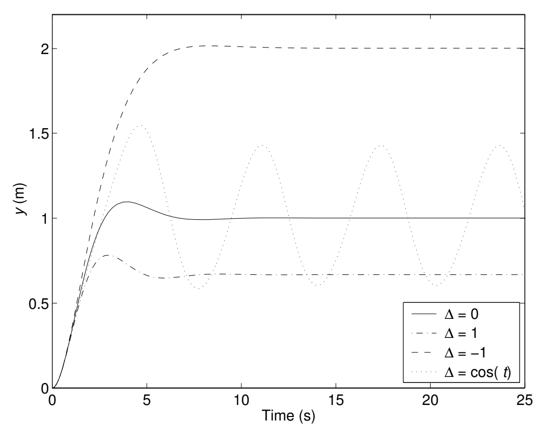

Figure 3: Open-loop responses

Hence, our objective is to minimize the parametric sensitivity of the system. The nominal system response obtained for $\Delta = 0$ in Figure 3 is desired to be preserved for all admissible parameter values and for any parameter rate of variation.

The dynamic behavior of this system can be kept as close as possible to a desired one by incorporating to its model (22) a channel (w, z) defined by the input and output matrices in (13) and (14), respectively, and computing an LPV controller (4) that minimizes the H_2 performance variable ν of the linear operator T_{wz} mapping w into z.

The first step is to choose real matrices $\mathcal{H}$, A_d, R_0 and R_1 that define the output matrix (14) and the criterion (12) to be optimized. Notice that the time domain specifications can be completely defined from the dynamic matrix of the stationary system (21) operating at the central point $\Delta(t) = 0$. The fact that the dynamic behavior of this LTI system is characterized by a pair of well-damped complex-conjugate poles $(s_{1,2} := -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2} = -0.6 \pm j0.8)$ led us to select $\mathcal{H} = I_2$ and a matrix A_d having the same natural frequency ($\omega_n = 1$) and damping factor ($\zeta = 0.6$),

$$A_d = \begin{bmatrix} 0 & 1 \\ -\omega_n^2 & -2\zeta\omega_n \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & -1.2 \end{bmatrix}.$$

Suitable and reasonable values for rise time, settling time, and peak overshoot are reflected in the matrices A_d and $\mathcal{H}$. However, the complete characterization of the criterion (12) requires that the ratio between the weighting matrices R_0 and R_1 be stipulated. By fixing the error weight $R_0 = I_2$ and performing the Algorithm 4.1 for different values of R_1, we obtain LPV controllers with performance properties presented in Table 1 and illustrated in Figure 4. Three main effects of the decrease of R_1 are immediately observed: the input energy increases, the optimal value ν decreases, and the dynamic behavior of the system tends to the desired one. These properties mean that the matrices A_d and $\mathcal{H}$, that dictate the central design specifications, have been appropriately chosen, with the former one indicating that the additional freedom R_1 penalizes adequately the input energy. These results validate the H_2 formulation of the LFT/LPV IMF problem derived in Section 3.

Finally, for $R_1 = 0.001$ the synthesized LPV controller is described by the following continuous-time state-space data and gain-scheduling function:

$$\begin{bmatrix} \dot{x}_K \\ u \\ z_K \end{bmatrix} = \begin{bmatrix} -1.4557 & -0.00955 & 0 \\ 0.00955 & 0 & 0.000595 \\ 0 & 917.73 & 0 \end{bmatrix} \begin{bmatrix} x_K \\ y \\ w_K \end{bmatrix},$$

$$\Delta_K(\Delta(t)) = -0.9151\alpha_1(\Delta(t)) + 0.9151\alpha_2(\Delta(t)).$$

It must be emphasized that a fast mode of the controller has been approximated by a static gain in the form of a feedthrough scalar, resulting in the first-order state description presented above and used for generating the corresponding simulation in Figure 4. We also recall that the coefficients $\alpha_{1,2}(t)$ are the polytopic coordinates of $\Delta(t)$ available in real time. This gain-scheduled controller is theoretically capable to keep the closed-loop response very close to the central ($\Delta = 0$) open-loop response for all admissible parameter trajectories, that is $\forall\Delta \in [-1, 1]$ and any rate of variation $d\Delta/dt$. We must keep in mind, however, that practical limit exists for online adjustment of the controller data and consequently, only realistic trajectories $\Delta(t)$ can be compensated by our LPV controller.

Table 1: Optimized H_2 performance for different values of the input weight

R_1	0.001	0.1	0.5	1.0
$\sqrt{\nu}$	0.021	0.203	0.410	0.525

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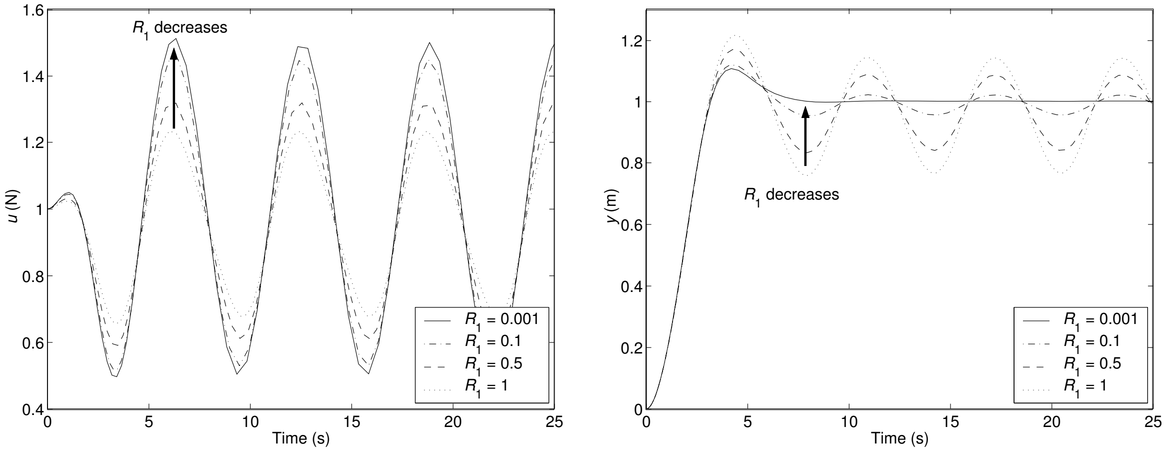

(a) - *Control,* $u(t)$ (b) - *Measure,* $y(t)$

Figure 4: Closed-loop responses to a unit step for different input weights and for $\Delta(t) = \cos(t)$

6 Conclusions

The optimal IMF problem has been reconsidered and treated in the context of LFT/LPV and H_2/H_∞ theories. The proposed synthesis procedure allow to attain adequate transient behaviors for LPV systems and is a practically valid alternative to pole-based methods for LTI systems:

- the IMF criterion can be combined with a rich variety of other closed-loop specifications in time or frequency domains when interpreted as a set of different H_2/H_∞ criteria;
- balancing these design requirements is carried out in a very natural way within the proposed design framework and conservatism is kept reasonable thanks to the use of different Lyapunov and scaling variables for each channel/specification;
- the H_2 specification corresponding to the IMF criterion does not impact the controller order;
- time-domain specifications for non-stationary systems are readily considered in the controller synthesis, overcoming difficulties concerning the lack of frequency-domain concepts for non-stationary systems and avoiding the tedious task of weight selection.

Finally, an illustrative example has been used to validate the proposed LFT/LPV IMF formulation. A more realistic LPV control application considering extra H_2 and/or H_∞ performance constraints that translate other desired properties like more stringent control limitations and robustness as well as studies about the influence of the bilinear transformations on the designed controller are subject of future investigations.

References

[1] P. Apkarian, P. C. Pellanda, and H. D. Tuan, *Mixed H_2/H_∞ Multi-Channel Linear Parameter-Varying Control in Discrete Time*, Syst. Contr. Letters **41** (2000), 333–346.

[2] S. Boyd, L. E. Ghaoui, E. Feron, and V. Balakrishnan, *Linear Matrix Inequalities in Systems and Control Theory*, vol. 15, SIAM Studies in Applied Mathematics, Philadelphia, 1994.

[3] M. Chilali, P. Gahinet, and P. Apkarian, *Robust Pole Placement in LMI Regions*, IEEE Trans. Automat. Contr. **44** (1999), no. 12, 2257–2270.

[4] M. C. de Oliveira, J. Bernussou, and J. C. Geromel, *A New Discrete-Time Robust Stability Condition*, Syst. Contr. Letters **37** (1999), 261–265.

[5] P. Gahinet, A. Nemirovski, A. J. Laub, and M. Chilali, *The LMI Control Toolbox for Use with Matlab*, The MathWorks Inc., 1994.

[6] E. A. Jonckheere and G. R. Yu, *Propulsion control of crippled aircraft by H-infinity model matching*, IEEE Trans. Contr. Syst. Technology **7** (1999), no. 2, 142–159.

[7] E. Kreindler and D. Rothschild, *Model Following in Linear-Quadratic-Optimization*, J. of Guidance, Control and Dynamics **14** (1976), no. 7, 835–842.

[8] J. C. Morris, P. Apkarian, and J. C. Doyle, *Synthesizing Robust Mode Shapes with μ and Implicit Model Following*, in Proc. IEEE Conf. on Control Applications, Dayton, Ohio **Sept.** (1992).

[9] P. C. Pellanda, P. Apkarian, and H. D. Tuan, *Missile Autopilot Design Via a Multi-Channel LFT/LPV Control Method*, Int J. Robust and Nonlinear Control **12** (2002), no. 1, 1–20.

[10] J. K. Shiau, G. N. Taranto, J. H. Chow, and H. A. Othman, *A decentralized Control Design Method For Model-Matching Robustness Problems*, Int. J. Contr. **68** (1997), no. 2, 401–416.

[11] G. N. Taranto, J. H. Chow, and H. A. Othman, *Robust Redesign of Power-System Damping Controllers*, IEEE Trans. Contr. Systems Technology **3** (1995), no. 3, 290–298.