Robust Filtering for Uncertain Nonlinearly Parameterized Plants

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***Abstract*—In this paper, we address the robust filtering problem for a wide class of systems whose state-space data assume a very general nonlinear dependence in the uncertain parameters. Our resolution methods rely on new linear matrix inequality characterizations of $\mathcal{H}_2$ and $\mathcal{H}_\infty$ performances, which, in conjunction with suitable linearization transformations of the variables, give rise to practical and computationally tractable formulations for the robust filtering problem.**

***Index Terms*—Linear matrix inequality (LMI), nonlinear parameterization, robust filtering.**

I. Introduction

THROUGHOUT this paper, we consider the uncertain linear system in the nonlinear fractional transformation (NFT) format

$$\begin{bmatrix} \dot{x}(t) \\ y(t) \\ z_\Delta(t) \\ z(t) \end{bmatrix} = \begin{bmatrix} A(\alpha) & B_\Delta(\alpha) & B(\alpha) \\ C(\alpha) & D_\Delta(\alpha) & D(\alpha) \\ C_\Delta(\alpha) & D_{\Delta z}(\alpha) & D_z(\alpha) \\ L(\alpha) & D_{\Delta\Delta}(\alpha) & M(\alpha) \end{bmatrix} \begin{bmatrix} x(t) \\ w_\Delta(t) \\ w(t) \end{bmatrix}$$
$$w_\Delta(t) = \Delta(\alpha) z_\Delta(t) \quad (1)$$

where $A(\alpha) \in \boldsymbol{R}^{n\times n}$, $B_\Delta(\alpha) \in \boldsymbol{R}^{n\times m_\Delta}$, $B(\alpha) \in \boldsymbol{R}^{n\times m}$, $D(\alpha) \in \boldsymbol{R}^{p\times m}$, $C_\Delta(\alpha) \in R^{m_\Delta \times n}$, $L(\alpha) \in \boldsymbol{R}^{q\times n}$, and $x \in \boldsymbol{R}^n$ is the state, $y \in \boldsymbol{R}^p$ is the measured output, $z \in \boldsymbol{R}^q$ is the output to be estimated, $w \in R^m$ is the disturbance, and $w_\Delta \in R^{m_\Delta}$ and $z_\Delta \in R^{m_\Delta}$ are introduced to materialize the uncertainty component of the system. The uncertain parameter α is assumed to evolve in the unit simplex Γ

$$\Gamma := \left\{ (\alpha_1, \ldots, \alpha_s) \colon \sum_{j=1}^{s} \alpha_j = 1, \, \alpha_j \geq 0 \right\}.$$

The state-space data in (1) is assumed linear in the parameter α, that is

$$\begin{bmatrix} A(\alpha) & B_\Delta(\alpha) & B(\alpha) \\ C(\alpha) & D_\Delta(\alpha) & D(\alpha) \\ C_\Delta(\alpha) & D_{\Delta z}(\alpha) & D_z(\alpha) \\ L(\alpha) & D_{\Delta\Delta}(\alpha) & M(\alpha) \\ 0 & \Delta(\alpha) & 0 \end{bmatrix} = \sum_{j=1}^{s} \alpha_j \begin{bmatrix} A_j & B_{\Delta j} & B_j \\ C_j & D_{\Delta j} & D_j \\ C_{\Delta j} & D_{\Delta z j} & D_{zj} \\ L_j & D_{\Delta\Delta j} & M_j \\ 0 & \Delta_j & 0 \end{bmatrix}. \quad (2)$$

Note that an equivalent representation of system (1) is the NFT

$$\begin{bmatrix} \dot{x}(t) \\ y(t) \\ z(t) \end{bmatrix} = \left(\begin{bmatrix} A(\alpha) & B(\alpha) \\ C(\alpha) & D(\alpha) \\ L(\alpha) & M(\alpha) \end{bmatrix} + \begin{bmatrix} B_\Delta(\alpha) \\ D_\Delta(\alpha) \\ D_{\Delta\Delta}(\alpha) \end{bmatrix} \Delta(\alpha) \times (I - D_{\Delta z}(\alpha)\Delta(\alpha))^{-1} \left[C_\Delta(\alpha) \quad D_z(\alpha) \right] \right) \times \begin{bmatrix} x(t) \\ w(t) \end{bmatrix}. \quad (3)$$

Clearly, the uncertain parameter α enters the system representation (3) in a highly nonlinear manner. This is in stark contrast with the linear parameter dependence of polytopic representations [8], [14], [17]. Obviously, any polytopic system is also a particular case of (1) or (3) with $\Delta(\alpha) = 0$. Alternatively, the NFT system (1) can be transformed into a standard linear fractional transformation (LFT) representation [18], where Δ only is allowed to depend on uncertain parameters. This amounts to augmenting the dimension of the uncertainty channel w_Δ, z_Δ. However, it is our opinion that this alternative representation dramatically deteriorates the performance of practical solution methods, as illustrated in Section V.

The filtering problem for the uncertain system (1) consists of constructing an estimator or "filter" in the form

$$\begin{bmatrix} \dot{x}_F(t) \\ z_F(t) \end{bmatrix} = \begin{bmatrix} \mathbf{A}_F & \mathbf{B}_F \\ \mathbf{L}_F & 0 \end{bmatrix} \begin{bmatrix} x_F(t) \\ y(t) \end{bmatrix}$$
$$\mathbf{A}_F \in \boldsymbol{R}^{n\times n}, \, \mathbf{L}_F \in \boldsymbol{R}^{q\times n} \quad (4)$$

which provides good robust estimation of the output z in (1). In the present paper, such a good estimation is based on the mixed $\mathcal{H}_2/\mathcal{H}_\infty$ criterion

$$\max_{\alpha\in\Gamma} \rho \|T(w, z - z_F)\|_{[2]} + (1-\rho)\|T(w, z - z_F)\|_\infty \to \min \quad (5)$$

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where $T(w, z-z_F)$ denotes the transfer function from the input signal w to the error signal $z - z_F$. The notation $\|.\|_{[2]}$ designates the generalized H_2 norm, whereas $\|.\|_\infty$ designates the H_∞ norm. The scalar ρ satisfies $0 \le \rho \le 1$ and plays the role of a trade-off coefficient. The meaning of these norms is further clarified in the sequel of the paper.

The robust filtering for this general uncertain systems has not been considered in the literature so far. A particular case of the $\mathcal{H}_2$ (Kalman) filtering for polytopic systems has been addressed in [8] and [14] by using the common Lyapunov function approach and in [17] by using the much less conservative parameter-dependent Lyapunov function approach. The mixed $\mathcal{H}_2/\mathcal{H}_\infty$ filtering for linear nominal systems (with no parameter α) has been particularly investigated in [9], [12], and [15] with different approaches and applications. In this paper, we propose a novel approach to handle the filtering problem where both parameter-dependent Lyapunov functions and parameter-dependent multipliers are utilized. Namely, our purpose is twofold.

- We introduce new linear matrix inequality (LMI) characterizations for the $\mathcal{H}_2$ and the $\mathcal{H}_\infty$ performances in the context of uncertain NFT systems. The currently known LMI characterizations are potentially conservative in the sense that they use a common Lyapunov function, regardless of the parameter values. With our new LMI characterizations, this weakness is partially eliminated.
- We establish new LMI-based techniques for the above robust mixed $\mathcal{H}_2/\mathcal{H}_\infty$ filtering problems. In addition, as a byproduct, a new method for the mixed $\mathcal{H}_2/\mathcal{H}_\infty$ filtering for the nominal case is derived, which, according to experiments, is much less conservative than the results in [9].

Note that the optimization formulation in [10] for the $\mathcal{H}_\infty$-filter of a particular class of LFT requires solving a nonlinear matrix inequality in the decision variables and, thus, does not provide a practical technique in general.

The structure of the paper is as follows. Section II discusses equivalent LMI characterizations of performances that will be used throughout the paper. These characterizations for the $\mathcal{H}_2$ and $\mathcal{H}_\infty$ norms of NFT systems are introduced in Section III and exploited in Section IV for filtering problems. Numerical tests and comparisons validating the proposed methods are given in Section V. Finally, an Appendix provides a proof of the central result of Section II.

The notation throughout the paper is fairly standard. M^T is the transpose of the matrix M, whereas $\mathcal{N}_M$ is any basis of its null space. For symmetric matrices, $M - N < 0$ ($M - N > 0$, respectively) means that $M - N$ is negative definite (positive definite, respectively). In symmetric block matrices or long matrix expressions, we use $*$ as an ellipsis for terms that are induced by symmetry, e.g.,

$$(*)\begin{bmatrix} S+(*) & * \\ M & Q \end{bmatrix} K^T \equiv K \begin{bmatrix} S+S^T & M^T \\ M & Q \end{bmatrix} K^T.$$

In addition, in long matrix expressions involving matrix functions of the parameter α, we use the shorthand

$$\begin{bmatrix} M_{11} & * \\ M_{12} & M_{22} \end{bmatrix}(\alpha) \equiv \begin{bmatrix} M_{11}(\alpha) & M_{12}^T(\alpha) \\ M_{12}(\alpha) & M_{22}(\alpha) \end{bmatrix}. \tag{6}$$

When there is a possibility of ambiguity, we use, for instance, I_n, 0_{nm} to indicate the dimensions of matrices. The boldface capital letters such as $\mathbf{X}$, $\mathbf{K}$, $\mathbf{R}$, etc., are used to emphasize matrix variables.

A. Useful Tools

Below, we recall a number of technical tools that are useful in the derivations.

- *Congruence transformation of matrices*: The matrix M is negative definite (positive definite, respectively) if and only if T^TMT is negative definite (positive definite, respectively) for any nonsingular matrix T of appropriate dimension. The matrix T^TMT is called congruent to M via the congruence transformation T.
- *Schur's complement formulas*:

$$\begin{bmatrix} M_{11} & M_{12} \\ M_{12}^T & M_{22} \end{bmatrix} < 0 \Leftrightarrow M_{22} < 0, \quad M_{11} - M_{12}M_{22}^{-1}M_{12}^T < 0$$

$$\Leftrightarrow M_{11} < 0, \quad M_{22} - M_{12}^TM_{11}^{-1}M_{12} < 0$$

for any matrices M_{11}, M_{12}, M_{22} of appropriate dimensions.

- *Projection lemma* [6]: Given a symmetric matrix $\Psi \in \boldsymbol{R}^{m\times m}$ and two matrices P, Q of column dimension m, the LMI problem

$$\Psi + P^T\mathbf{X}^TQ + Q^T\mathbf{X}P < 0$$

is solvable with respect to $\mathbf{X}$ of compatible dimension if and only if

$$\mathcal{N}_P^T\Psi\mathcal{N}_P < 0, \quad \text{and } \mathcal{N}_Q^T\Psi\mathcal{N}_Q < 0.$$

- *Linearly parameterized matrix inequality (LPMI) over the unit simplex* Γ: The parameterized inequality

$$\sum_{j=1}^{s} \alpha_j\mathcal{L}_j(\mathbf{P}) < 0, \qquad \forall\, \alpha \in \Gamma \tag{7}$$

is feasible in the decision variable $\mathbf{P}$ if and only if the following system of matrix inequalities is feasible in $\mathbf{P}$:

$$\mathcal{L}_j(\mathbf{P}) < 0, \qquad j = 1, 2, \ldots, s. \tag{8}$$

Here, $\mathcal{L}_j(\mathbf{P})$ are arbitrary matrix-valued functions of $\mathbf{P}$.

II. Auxiliary Results

As it is well known, a major advantage of the LMI approach in comparison to classical techniques is to provide additional flexibility to tackle a wide range of challenging problems such as multiobjective controls, robust control with real uncertain parameters, linear parameter-varying control, etc. An important restriction, however, is that a single parameter-free Lyapunov function is used for checking the system performances. Such a drawback entails conservativeness of solutions and often limits the practical appeal of LMI methods.

For discrete-time systems, this weakness has been partly eliminated in [4] and [11]. For linear continuous systems, a genuine extension for stability analysis and $\mathcal{H}_2$ performances has been proposed in [3] and [17], where the Lyapunov matrix and system matrices containing design parameters are to some extend separated. However, extensions to NFT systems in (1) remain challenging. In this section, we describe some alternative LMI formulations that are revealed to be very practical for the robust filtering problem.

Theorem 1:

a) The LMI

$$\text{i)}\quad \mathbf{X} > 0, \begin{bmatrix} A^T\mathbf{X} + \mathbf{X}A & \mathbf{X}B & C^T \\ B^T\mathbf{X} & Q_{11} & Q_{12} \\ C & Q_{12}^T & Q_{22} \end{bmatrix} < 0 \tag{9}$$

is feasible the decision variable $\mathbf{X}$ if and only if there is scalar $\mu > 0$ such that either one of the LMIs, which are as in (10)–(13), shown at the bottom of the page, is feasible in the decision variables $\mathbf{X}$, $\mathbf{V}$ and $\mathbf{V}_1$.

b) When $C = 0$, the feasibility of (9) in $\mathbf{X}$ is equivalent to the feasibility of (12) in $\mathbf{X}$, $\mathbf{V}$ for $\mu = 1$, i.e., the feasibility in $\mathbf{X}$, $\mathbf{V}$ of the LMI is as in (14), shown at the bottom of the page.

See the Appendix for proofs.

Remark: As compared with (9), the advantage of the formulations (10)–(14) is that the Lyapunov variable $\mathbf{X}$ is, to some extent, separated from the data matrices A, B, C usually containing the design variables. This will yield additional freedom for using different Lyapunov variables $\mathbf{X}$ associated with various specifications. LMI (11) is useful for the analysis problem but not for the synthesis purpose because it involves two slack variables $\mathbf{V}$ and $\mathbf{V}_1$ that render the linearization of the problem a difficult task. The form in (13) has proved to be the most useful in our filtering context. When $C = 0$, however, one should definitely use the simple form in (14).

$$\text{ii)}\quad \mathbf{X} > 0, \begin{bmatrix} A^T\mathbf{V} + \mathbf{V}^TA & \mathbf{V}B & C^T & \mathbf{X} - \mathbf{V}^T + \mu A^T\mathbf{V} \\ B^T\mathbf{V} & Q_{11} & Q_{12} & \mu B^T\mathbf{V} \\ C & Q_{12}^T & Q_{22} & 0 \\ \mathbf{X} - \mathbf{V} + \mu\mathbf{V}^TA & \mu\mathbf{V}^TB & 0 & -2\mu(\mathbf{V} + \mathbf{V}^T) \end{bmatrix} < 0 \tag{10}$$

$$\text{iii)}\quad \mathbf{X} > 0, \begin{bmatrix} A^T\mathbf{V} + \mathbf{V}^TA & \mathbf{V}^TB & C^T & \mathbf{X} - \mathbf{V}^T + A^T\mathbf{V}_1 \\ B^T\mathbf{V} & Q_{11} & Q_{12} & B^T\mathbf{V}_1 \\ C & Q_{12}^T & Q_{22} & 0 \\ \mathbf{X} - \mathbf{V} + \mathbf{V}_1^TA & \mathbf{V}_1^TB & 0 & -(\mathbf{V}_1 + \mathbf{V}_1^T) \end{bmatrix} < 0 \tag{11}$$

$$\text{iv)}\quad \begin{bmatrix} -(\mathbf{V} + \mathbf{V}^T) & \mathbf{V}^TA + \mathbf{X} & \mathbf{V}^TB & 0 & \mathbf{V}^T \\ A^T\mathbf{V} + \mathbf{X} & -\mu\mathbf{X} & 0 & C^T & 0 \\ B^T\mathbf{V} & 0 & Q_{11} & Q_{12} & 0 \\ 0 & C & Q_{12}^T & Q_{22} & 0 \\ \mathbf{V} & 0 & 0 & 0 & -\mathbf{X}/\mu \end{bmatrix} < 0 \tag{12}$$

$$\text{v)}\quad \begin{bmatrix} -\mu(\mathbf{V} + \mathbf{V}^T) & \mathbf{V}^TA + \mathbf{X} & \mathbf{V}^TB & 0 & \mu\mathbf{V}^T \\ A^T\mathbf{V} + \mathbf{X} & -\mathbf{X} & 0 & C^T & 0 \\ B^T\mathbf{V} & 0 & Q_{11} & Q_{12} & 0 \\ 0 & C & Q_{12}^T & Q_{22} & 0 \\ \mu\mathbf{V} & 0 & 0 & 0 & -\mathbf{X} \end{bmatrix} < 0 \tag{13}$$

$$\text{vi)}\quad \mathbf{X} > 0, \begin{bmatrix} -(\mathbf{V} + \mathbf{V}^T) & \mathbf{V}^TA + \mathbf{X} & \mathbf{V}^TB & 0 & \mathbf{V}^T \\ A^T\mathbf{V} + \mathbf{X} & -\mathbf{X} & 0 & 0 & 0 \\ B^T\mathbf{V} & 0 & Q_{11} & Q_{12} & 0 \\ 0 & 0 & Q_{12}^T & Q_{22} & 0 \\ \mathbf{V} & 0 & 0 & 0 & -\mathbf{X} \end{bmatrix} < 0 \tag{14}$$

III. LMI Characterizations for Norm Constraints

A. Symmetric Scaling in NFT

We first note that it is possible to rewrite the overall system (1) and (4) with the error signal $z_{cl} = z - z_F$ explicitly as

$$\begin{bmatrix} \dot{x}_{cl}(t) \\ z_\Delta(t) \\ z_{cl}(t) \end{bmatrix} = \begin{bmatrix} \mathcal{A}_{cl}(\alpha) & \mathcal{B}_{\Delta cl}(\alpha) & \mathcal{B}_{cl}(\alpha) \\ \mathcal{C}_\Delta(\alpha) & D_{\Delta z}(\alpha) & D_z(\alpha) \\ \mathcal{L}_{cl}(\alpha) & D_{\Delta\Delta}(\alpha) & M(\alpha) \end{bmatrix} \begin{bmatrix} x_{cl}(t) \\ w_\Delta(t) \\ w(t) \end{bmatrix}$$
$$w_\Delta = \Delta(\alpha) z_\Delta \tag{15}$$

where

$$x_{cl} = \begin{bmatrix} x \\ x_F \end{bmatrix}, \quad \mathcal{A}_{cl}(\alpha) = \begin{bmatrix} A(\alpha) & 0 \\ \mathbf{B}_F C(\alpha) & \mathbf{A}_F \end{bmatrix}$$
$$\mathcal{B}_{\Delta cl}(\alpha) = \begin{bmatrix} B_\Delta(\alpha) \\ \mathbf{B}_F D_\Delta(\alpha) \end{bmatrix}, \quad \mathcal{B}_{cl}(\alpha) = \begin{bmatrix} B(\alpha) \\ \mathbf{B}_F D(\alpha) \end{bmatrix}$$
$$\mathcal{C}_\Delta(\alpha) = [\, C_\Delta(\alpha) \quad 0 \,], \quad \mathcal{L}_{cl}(\alpha) = [\, L(\alpha) \quad -\mathbf{L}_F \,]. \tag{16}$$

In order to characterize the relationships between w_Δ and z_Δ, we will use a specific class of scalings already introduced in [13] (see also [4])

$$z_\Delta^T \mathbf{R}_i(\alpha) z_\Delta + w_\Delta^T \mathbf{S}_i(\alpha) w_\Delta \geq 0, \qquad \mathbf{S}_i(\alpha) < 0, \; i = 1, 2 \tag{17}$$

$$z_\Delta^T \mathbf{R}(\alpha) z_\Delta + w_\Delta^T \mathbf{S}(\alpha) w_\Delta \geq 0, \qquad \mathbf{S}(\alpha) < 0 \tag{18}$$

for all nonzero w_Δ, z_Δ satisfying (15) and $\alpha \in \Gamma$. Substituting $w_\Delta = \Delta(\alpha) z_\Delta$ into (18) yields

$$\begin{aligned} (18) \Leftrightarrow \; & z_\Delta^T(\mathbf{R}(\alpha) + \Delta^T(\alpha)\mathbf{S}(\alpha)\Delta(\alpha)) z_\Delta \geq 0, \quad \forall z_\Delta \\ \Leftrightarrow \; & (\mathbf{R} + \Delta^T \mathbf{S} \Delta)(\alpha) \geq 0 \\ \Leftrightarrow \; & \begin{bmatrix} \mathbf{R} & \Delta^T \\ \Delta & -\mathbf{S}^{-1} \end{bmatrix}(\alpha) \geq 0 \quad \text{(by Schur's complement).} \end{aligned} \tag{19}$$

Analogously, (17) is equivalent to

$$\begin{bmatrix} \mathbf{R}_i & \Delta^T \\ \Delta & -\mathbf{S}_i^{-1} \end{bmatrix}(\alpha) \geq 0, \qquad i = 1, 2. \tag{20}$$

Choosing a linear parameter dependence in the form

$$\begin{bmatrix} \mathbf{R}_i \\ \mathbf{S}_i \\ \mathbf{S} \end{bmatrix}(\alpha) = \sum_{j=1}^{s} \alpha_j \begin{bmatrix} \mathbf{R}_{ij} \\ \mathbf{S}_{ij} \\ \mathbf{S}_j \end{bmatrix} \tag{21}$$

and according to the inequalities

$$\begin{aligned} -\mathbf{S}_i(\alpha)^{-1} &\geq \mathbf{H}_i^T \mathbf{S}_i(\alpha) \mathbf{H}_i + (\mathbf{H}_i + \mathbf{H}_i^T) \quad \forall \mathbf{H}_i \\ -\mathbf{S}(\alpha)^{-1} &\geq \mathbf{G}^T \mathbf{S}(\alpha) \mathbf{G} + (\mathbf{G} + \mathbf{G}^T) \quad \forall \mathbf{G} \end{aligned} \tag{22}$$

it is not difficult to see that $\mathbf{R}_i(\alpha)$, $\mathbf{S}_i(\alpha)$, $\mathbf{R}(\alpha)$, $\mathbf{S}(\alpha)$ satisfy (20), (19) if there are matrices $\mathbf{H}_i$, $\mathbf{G}$ of the same dimension such that

$$\begin{bmatrix} \mathbf{R}_{ij} & \Delta_j^T \mathbf{H}_i^T \\ \mathbf{H}_i \Delta_j & \mathbf{S}_{ij} + (\mathbf{H}_i + \mathbf{H}_i^T) \end{bmatrix} \geq 0, \quad i = 1, 2; \; j = 1, 2 \ldots, s \tag{23}$$

$$\begin{bmatrix} \mathbf{R}_j & \Delta_j^T \mathbf{G}^T \\ \mathbf{G}\Delta_j & \mathbf{S}_j + (\mathbf{G} + \mathbf{G}^T) \end{bmatrix} \geq 0, \quad j = 1, 2 \ldots, s. \tag{24}$$

B. $\mathcal{H}_2$-Norm Characterization

In this subsection, we assume that

$$\begin{aligned} M(\alpha) &= 0 \\ D_{\Delta\Delta}(\alpha)\Delta(\alpha)(I - D_{\Delta z}(\alpha)\Delta(\alpha))^{-1} D_z(\alpha) &= 0 \end{aligned} \tag{25}$$

and consider the $\mathcal{H}_2$-norm characterization for system (15).

Assume that $\mathbf{R}_i(\alpha)$, $\mathbf{S}_i(\alpha)$ satisfy (20), i.e., (17) holds true. If there are matrices $\mathbf{X}(\alpha) > 0$ and $\mathbf{Z}(\alpha) > 0$ such that

$$\frac{d}{dt}[x_{cl}^T(t)\mathbf{X}(\alpha)x_{cl}(t)] + z_\Delta^T(t)\mathbf{R}_1(\alpha)z_\Delta(t) + w_\Delta^T(t)\mathbf{S}_1(\alpha)w_\Delta(t) - \|w(t)\|^2 < 0 \tag{26}$$

$$\frac{1}{\nu} z_{cl}^T(t) z_{cl}(t) + z_\Delta^T(t)\mathbf{R}_2(\alpha)z_\Delta(t) + w_\Delta^T(t)\mathbf{S}_2(\alpha)w_\Delta(t) - [x_{cl}^T(t)\mathbf{X}(\alpha)x_{cl}(t)] < 0 \tag{27}$$

then, for all w_Δ, z_Δ satisfying (15), we have

$$\frac{d}{dt}[x_{cl}^T(t)\mathbf{X}(\alpha)x_{cl}(t)] - \|w(t)\|^2 < 0 \tag{28}$$

$$\frac{1}{\nu} z_{cl}^T(t) z_{cl}(t) - x_{cl}^T(t)\mathbf{X}(\alpha)x_{cl}(t) < 0. \tag{29}$$

The latter inequalities lead to

$$x_{cl}^T(t)\mathbf{X}(\alpha)x_{cl}(t) < \int_0^t \|w(s)\|^2 \, ds$$
$$z_{cl}^T(t) z_{cl}(t) < \nu x_{cl}^T(t)\mathbf{X}(\alpha)x_{cl}(t) < \nu \int_0^t \|w(s)\|^2 \, ds$$

implying

$$\begin{aligned} \sup_t \|z_{cl}(t)\|^2 &< \nu \int_0^{+\infty} \|w(t)\|^2 \, dt \\ &:= \nu \|w\|_2^2, \qquad \forall w \in L_2 \end{aligned} \tag{30}$$

that is, the $\mathcal{H}_2$-norm of system (43) is less than ν.

Now, rewriting the left-hand side of inequalities (26) and (27) as quadratic functionals in (x, w, w_Δ) and by a Schur's complement argument, the following result is obtained.

Lemma 1: One has

$$\max_{\alpha \in \Gamma} \|T(w, z_{cl})\|_{[2]}^2 < \nu \tag{31}$$

if there are symmetric matrix $\mathbf{X}(\alpha) > 0$ and scalings $\mathbf{R}_i(\alpha)$, $\mathbf{S}_i(\alpha)$ satisfying (20), and moreover

$$\begin{bmatrix} \mathcal{A}_{cl}^T \mathbf{X} + \mathbf{X}\mathcal{A}_{cl} & * & * \\ \begin{bmatrix} \mathcal{B}_{\Delta cl}^T \\ \mathcal{B}_{cl}^T \end{bmatrix} \mathbf{X} & \begin{bmatrix} \mathbf{S}_1 & * \\ 0 & -I \end{bmatrix} & * \\ \mathcal{C}_\Delta & [\, D_{\Delta z} \quad D_z \,] & -\mathbf{R}_1^{-1} \end{bmatrix}(\alpha) < 0$$
$$\forall \alpha \in \Gamma \tag{32}$$

$$\begin{bmatrix} -\mathbf{X} & * & * & * \\ 0 & \mathbf{S}_2 & * & * \\ \mathcal{C}_\Delta & D_{\Delta z} & -\mathbf{R}_2^{-1} & * \\ \mathcal{L}_{cl} & D_{\Delta\Delta} & 0 & -\nu I \end{bmatrix}(\alpha) < 0, \qquad \forall\, \alpha \in \Gamma. \tag{33}$$

□

Thanks to Theorem 1, one can provide an alternative form of the $\mathcal{H}_2$ performance that facilitates tractability of the robust filtering problem.

Theorem 2: The feasibility of inequalities (20), (32), and (33) with respect to $\mathbf{X}(\alpha)$ and $\mathbf{S}_i(\alpha)$, $\mathbf{R}_i(\alpha)$ defined in (21) characterizing the performance bound in (31) is equivalent to the existence of $\mu > 0$ such that inequality (23) and the inequalities (34) and (35), shown at the bottom of the page, are feasible in $\mathbf{X}(\alpha)$, $\mathbf{V}(\alpha)$, $\mathbf{S}_i(\alpha)$, $\mathbf{R}_i(\alpha)$, $\mathbf{F}_i(\alpha)$, and $\mathbf{H}_i$. □

C. $\mathcal{H}_\infty$-Norm

A very similar result can be established for the H_∞ performance. Consider $\mathbf{R}(\alpha)$, $\mathbf{S}(\alpha)$ satisfying (19) and $\mathbf{Y}(\alpha) > 0$ such that

$$\frac{d}{dt}\left[x_{cl}^T(t)\mathbf{Y}(\alpha)x_{cl}(t)\right] + z_\Delta^T(t)\mathbf{R}(\alpha)z_\Delta(t) + w_\Delta^T(t) \times \mathbf{S}(\alpha)w_\Delta(t) + \gamma^{-1}\|z_{cl}(t)\|^2 - \gamma\|w(t)\|^2 < 0. \tag{36}$$

Then

$$\frac{d}{dt}\left[x_{cl}^T(t)\mathbf{Y}(\alpha)x_{cl}(t)\right] + \gamma^{-1}\|z_{cl}(t)\|^2 - \gamma\|w(t)\|^2 < 0 \tag{37}$$

and hence

$$\int_0^T \|z_{cl}(t)\|^2 < \gamma^2 \int_0^T \|w(t)\|^2\, dt \tag{38}$$

for all z_Δ, w_Δ satisfying (15). This is equivalent to saying that the $\mathcal{H}_\infty$-norm of system (15) is less than γ. Again, rewriting the left-hand side of (36) as a quadratic functional in (x, w_Δ, w) and by a Schur's complement argument, the following result is obtained.

Theorem 3: The performance constraint

$$\max_{\alpha\in\Gamma} \|T(w, z_{cl})\|_\infty < \gamma \tag{39}$$

is satisfied whenever there are $\mathbf{Y}(\alpha) > 0$ and $\mathbf{R}(\alpha)$, $\mathbf{S}(\alpha)$ satisfying (19), and moreover

$$\begin{bmatrix} \mathcal{A}_{cl}^T\mathbf{Y} + \mathbf{Y}\mathcal{A}_{cl} & * & * & * \\ \begin{bmatrix} \mathcal{B}_{\Delta cl}^T \\ \mathcal{B}_{cl}^T \end{bmatrix}\mathbf{Y} & \begin{bmatrix} \mathbf{S} & * \\ 0 & -\gamma I \end{bmatrix} & * & * \\ \mathcal{C}_\Delta & [\,D_{\Delta z} \quad D_z\,] & -\mathbf{R}^{-1} & * \\ \mathcal{L}_{cl} & [\,D_{\Delta\Delta} \quad M\,] & 0 & -\gamma I \end{bmatrix} \times(\alpha) < 0, \qquad \forall\, \alpha \in \Gamma. \tag{40}$$

By virtue of Theorem 1, the feasibility of (19) and (40) in $\mathbf{Y}(\alpha)$, $\mathbf{S}(\alpha)$, $\mathbf{R}(\alpha)$ is equivalent to the feasibility of (24) and the inequalities in (41), shown at the bottom of the page, in $\mathbf{Y}(\alpha)$, $\mathbf{V}(\alpha)$, $\mathbf{F}(\alpha)$, $\mathbf{G}$, and $\mu > 0$. □

$$\begin{bmatrix} -\mu(\mathbf{V}+\mathbf{V}^T) & * & * & * & * \\ \mathcal{A}_{cl}^T\mathbf{V} + \mathbf{X} & -\mathbf{X} & * & * & * \\ \begin{bmatrix} \mathcal{B}_{\Delta cl}^T \\ \mathcal{B}_{cl}^T \end{bmatrix}\mathbf{V} & O & \begin{bmatrix} \mathbf{S}_1 & * \\ 0 & -I \end{bmatrix} & * & * \\ 0 & \mathbf{F}_1\mathcal{C}_\Delta & \mathbf{F}_1[\,D_{\Delta z} \quad D_z\,] & \mathbf{R}_1 - (\mathbf{F}_1 + \mathbf{F}_1^T) & * \\ \mu\mathbf{V} & 0 & 0 & 0 & -\mathbf{X} \end{bmatrix}(\alpha) < 0, \qquad \forall\, \alpha \in \Gamma \tag{34}$$

$$\begin{bmatrix} -\mathbf{X} & * & * & * \\ 0 & \mathbf{S}_2 & * & * \\ \mathbf{F}_2\mathcal{C}_\Delta & \mathbf{F}_2 D_{\Delta z} & \mathbf{R}_2 - (\mathbf{F}_2 + \mathbf{F}_2^T) & * \\ \mathcal{L}_{cl} & D_{\Delta\Delta} & 0 & -\nu I \end{bmatrix}(\alpha) < 0, \qquad \forall\, \alpha \in \Gamma \tag{35}$$

$$\begin{bmatrix} -\mu(\mathbf{V}+\mathbf{V}^T) & * & * & * & * & * \\ \mathcal{A}_{cl}^T\mathbf{V} + \mathbf{Y} & -\mathbf{Y} & * & * & * & * \\ \begin{bmatrix} \mathcal{B}_{\Delta cl}^T \\ \mathcal{B}_{cl}^T \end{bmatrix}\mathbf{V} & 0 & \begin{bmatrix} \mathbf{S} & * \\ 0 & -\gamma I \end{bmatrix} & * & * & * \\ 0 & \mathbf{F}\mathcal{C}_\Delta & \mathbf{F}[\,D_{\Delta z} \quad D_z\,] & \mathbf{R} - (\mathbf{F} + \mathbf{F}^T) & * & * \\ 0 & \mathcal{L}_{cl} & [\,D_{\Delta\Delta} \quad M\,] & 0 & -\gamma I & * \\ \mu\mathbf{V} & 0 & 0 & 0 & 0 & -\mathbf{Y} \end{bmatrix}(\alpha) < 0, \qquad \forall\, \alpha \in \Gamma \tag{41}$$

IV. Robust Filters for NFT

This section aims at developing a constructive method for the robust filtering problem. To this end, we systematically exploit the performance characterizations in (34) and (41). As clarified later in the text, this task becomes quite immediate by choosing parameter-independent matrices for V, F_i, and F:

$$\mathbf{V}(\alpha) \equiv \mathbf{V}, \quad \mathbf{F}_i(\alpha) \equiv \mathbf{F}_i, \quad \mathbf{F}(\alpha) \equiv \mathbf{F}, \qquad \forall\, \alpha \in \Gamma.$$

From now on, the following shorthand notations are used:

$$\Upsilon = \begin{bmatrix} 0_n \\ I_n \end{bmatrix}, \; \Theta = \begin{bmatrix} I_n \\ 0_n \end{bmatrix}, \; \Theta_p = \begin{bmatrix} I_p \\ 0_{np} \end{bmatrix}, \; \mathcal{I} = [\, I_n \quad I_n \,].$$

With the matrix definitions

$$\begin{aligned} \mathcal{B}_j &= [\, B_{\Delta j} \quad B_j \,], \quad \mathcal{C}_j = \begin{bmatrix} C_j & 0_{pn} \\ 0_n & I_n \end{bmatrix} \\ \mathcal{D}_j &= [\, D_{\Delta j} \quad D_j \,], \quad \mathcal{D}_{zj} = [\, D_{\Delta zj} \quad D_{zj} \,] \end{aligned} \tag{42}$$

we have

$$\begin{bmatrix} A_j & 0_n \\ 0_n & 0_n \end{bmatrix} = \Theta A_j \Theta^T, \quad [\, C_{\Delta j} \quad 0_{m_\Delta n} \,] = C_{\Delta j}\Theta^T$$

$$\begin{bmatrix} B_{\Delta j} & B_j \\ 0_{nm_\Delta} & 0_{nm} \end{bmatrix} = \Theta \mathcal{B}_j, \quad \begin{bmatrix} D_{\Delta j} & D_j \\ 0_{nm_\Delta} & 0_{nm} \end{bmatrix} = \Theta_p \mathcal{D}_j.$$

It follows that $\mathcal{A}_{cl}(\alpha)$, $\mathcal{B}_{\Delta cl}(\alpha)$, and $\mathcal{B}_{cl}(\alpha)$ defined in (16) can be rewritten as affine functions of the filter variable $\mathbf{K} = [\mathbf{B}_F \quad \mathbf{A}_F\,]$

$$\begin{aligned} &[\, \mathcal{A}_{cl} \mid \mathcal{B}_{\Delta cl} \quad \mathcal{B}_{cl} \,](\alpha) \\ &\quad = \sum_{j=1}^{s} \alpha_j \left([\, \Theta A_j \Theta^T \mid \Theta \mathcal{B}_j \,] + \Upsilon \mathbf{K} [\, \mathcal{C}_j \mid \Theta_p \mathcal{D}_j \,] \right). \end{aligned} \tag{43}$$

A. Robust $\mathcal{H}_2$ Filter

Choosing the parameter-dependent Lyapunov variable $\mathbf{X}(\alpha)$ in (34) and (35) as

$$\mathbf{X}(\alpha) = \sum_{j=1}^{s} \alpha_j \mathbf{X}_j \tag{44}$$

and $\mathbf{S}_i(\alpha)$, $\mathbf{R}_i(\alpha)$ according to (21), inequality (34) becomes an LPMI [see (7)] and therefore reduces to the finite set of inequalities as in (45), shown at the bottom of the next page. Examination of (45) reveals that there is only one bilinear term involving the filter variable $\mathbf{K}$ and the slack variable $\mathbf{V}$

$$\mathbf{K}^T \Upsilon^T \mathbf{V} = \begin{bmatrix} \mathbf{B}_F^T \\ \mathbf{A}_F^T \end{bmatrix} [\, \mathbf{V}_{21} \quad \mathbf{V}_{22} \,]$$

where $\mathbf{V}$ is partitioned as

$$\mathbf{V} = \begin{bmatrix} \mathbf{V}_{11} & \mathbf{V}_{12} \\ \mathbf{V}_{21} & \mathbf{V}_{22} \end{bmatrix}, \quad \mathbf{V}_{ij} \in R^{n \times n}. \tag{46}$$

With this in mind, the problem can be turned into a standard LMI program through the following procedure [4]:

- Define

$$\Pi_{\mathbf{V}} = \begin{bmatrix} I & 0 \\ 0 & \mathbf{V}_{22}^{-1}\mathbf{V}_{21} \end{bmatrix}. \tag{47}$$

- Introduce the auxiliary variables

$$\begin{aligned} \hat{\mathbf{V}} &= \begin{bmatrix} \hat{\mathbf{V}}_1 & \hat{\mathbf{V}}_2 \\ \hat{\mathbf{V}}_3 & \hat{\mathbf{V}}_3 \end{bmatrix} = \begin{bmatrix} \mathbf{V}_{11} & \mathbf{V}_{12}\mathbf{V}_{22}^{-1}\mathbf{V}_{21} \\ \mathbf{V}_{21}^T \mathbf{V}_{22}^{-T} \mathbf{V}_{21} & \mathbf{V}_{21}^T \mathbf{V}_{22}^{-T} \mathbf{V}_{21} \end{bmatrix} \\ &= \Pi_{\mathbf{V}}^T \mathbf{V} \Pi_{\mathbf{V}} \\ \hat{\mathbf{X}}_j &= \Pi_{\mathbf{V}}^T \mathbf{X}_j \Pi_{\mathbf{V}}, \quad \hat{\mathbf{K}} = [\, \hat{\mathbf{B}}_F \quad \hat{\mathbf{A}}_F \,] = \mathbf{V}_{21}^T \mathbf{K} \Pi_{\mathbf{V}} \\ &= [\, \mathbf{V}_{21}^T \mathbf{B}_F \quad \mathbf{V}_{21}^T \mathbf{A}_F \mathbf{V}_{22}^{-1} \mathbf{V}_{21} \,] \end{aligned} \tag{48}$$

  for which it is easily verified that

$$\begin{aligned} \Theta^T \mathbf{V} \Pi_{\mathbf{V}} &= [\, \hat{\mathbf{V}}_1 \quad \hat{\mathbf{V}}_2 \,] = \Theta^T \hat{\mathbf{V}} \\ \Pi_{\mathbf{V}} \Theta &= \Pi_{\mathbf{V}}^T \Theta = \Theta, \quad \Upsilon^T \mathbf{V} \Pi_{\mathbf{V}} = \mathbf{V}_{21} \mathcal{I} \end{aligned} \tag{49}$$

$$\Pi_{\mathbf{V}}^T \mathcal{C}_j^T = \mathcal{C}_j^T \Pi_{\mathbf{V}}^T, \quad \Pi_{\mathbf{V}}^T \mathbf{K}^T \Upsilon^T \mathbf{V} \Pi_{\mathbf{V}} = \hat{\mathbf{K}}^T \mathcal{I}. \tag{50}$$

- Perform in (45) the congruence transformation

$$\mathrm{diag}\, [\, \Pi_{\mathbf{V}} \quad \Pi_{\mathbf{V}} \quad I \quad I \quad \Pi_{\mathbf{V}} \,].$$

  This leads to the identities

$$\begin{aligned} \Pi_{\mathbf{V}}^T \Theta A_j^T \Theta^T \mathbf{V} \Pi_{\mathbf{V}} &= \Theta A_j^T \Theta^T \hat{\mathbf{V}} \\ \Pi_{\mathbf{V}}^T \mathcal{C}_j^T \mathbf{K}^T \Upsilon^T \mathbf{V} \Pi_{\mathbf{V}} &= \mathcal{C}_j^T \hat{\mathbf{K}}^T \mathcal{I} \\ \mathcal{B}_j^T \Theta^T \mathbf{V} \Pi_{\mathbf{V}} &= \mathcal{B}_j^T \Theta^T \hat{\mathbf{V}} \\ \mathbf{F}_1 C_{\Delta j} \Theta^T \Pi_{\mathbf{V}} &= \mathbf{F}_1 C_{\Delta j} \Theta^T \\ \mathcal{D}_j^T \Theta_p^T \mathbf{K}^T \Upsilon^T \mathbf{V} \Pi_{\mathbf{V}} &= \mathcal{D}_j^T \Theta_p^T \Pi_{\mathbf{V}}^T \mathbf{K} \Upsilon^T \mathbf{V} \Pi_{\mathbf{V}} \\ &= \mathcal{D}_j^T \Theta_p^T \hat{\mathbf{K}}^T \mathcal{I}. \end{aligned} \tag{51}$$

As a result, (45) is reduced to the inequality in (52), shown at the bottom of the next page, in $\hat{\mathbf{X}}_j$, $\mathbf{R}_{ij}$, $\mathbf{S}_{ij}$, $\hat{\mathbf{K}}$, $\hat{\mathbf{V}}$, $\mathbf{F}_1$, μ,

$$\begin{bmatrix} -\mu(\mathbf{V}+\mathbf{V}^T) & * & * & * & * \\ \Theta A_j^T \Theta^T \mathbf{V} + \mathcal{C}_j^T \mathbf{K}^T \Upsilon^T \mathbf{V} + \mathbf{X}_j & -\mathbf{X}_j & * & * & \\ \mathcal{B}_j^T \Theta^T \mathbf{V} + \mathcal{D}_j^T \Theta_p^T \mathbf{K}^T \Upsilon^T \mathbf{V} & 0 & \begin{bmatrix} \mathbf{S}_{1j} & * \\ 0 & -I_m \end{bmatrix} & * & * \\ 0 & \mathbf{F}_1 C_{\Delta j} \Theta^T & \mathbf{F}_1 \mathcal{D}_{zj} & \mathbf{R}_{1j} - (\mathbf{F}_1 + \mathbf{F}_1^T) & * \\ \mu \mathbf{V} & 0 & 0 & 0 & -\mathbf{X}_j \end{bmatrix} < 0$$

$$j = 1, 2, \ldots, s \tag{45}$$

which is nonlinear in the scalar $\mu > 0$ only. Thus, by using a line search in μ, we can check the feasibility of (52) by solving a sequence of LMI problems.

On the other hand, performing the congruence transformation

$$\mathrm{diag}\,[\,\Pi_{\mathbf{V}} \quad I \quad I \quad I\,]$$

in (35) and using the structure of $\mathcal{C}_\Delta(\alpha)$ in (16) leads to the following LMI:

$$\begin{bmatrix} -\hat{\mathbf{X}}_j & * & * & * \\ 0 & \mathbf{S}_{2j} & * & * \\ \mathbf{F}_2 C_{\Delta j}\Theta^T & \mathbf{F}_2 D_{\Delta zj} & \mathbf{R}_{2j} - (\mathbf{F}_2 + \mathbf{F}_2^T) & * \\ [\,L \quad -\hat{\mathbf{L}}_F\,] & D_{\Delta\Delta j} & 0 & -\nu I \end{bmatrix} < 0$$
$$j = 1, 2, \ldots, s \tag{53}$$

where

$$\hat{\mathbf{L}}_F = \mathbf{L}_F \mathbf{V}_{22}^{-1}\mathbf{V}_{21}. \tag{54}$$

Summing up, based on Theorem 2, we have established the following.

Theorem 4: There exists a filter (4) that satisfies the estimation condition (31) whenever there is $\mu > 0$ such that the LMI constraints (23), (52), and (53) are feasible in $\hat{\mathbf{V}}$, $\hat{\mathbf{X}}_j$, $\mathbf{S}_{ij}$, $\mathbf{R}_{ij}$, $\hat{\mathbf{K}}$, and $\hat{\mathbf{L}}_F$, $\mathbf{H}_i$, $\mathbf{F}_i$.

The matrix data $\mathbf{A}_F$, $\mathbf{B}_F$, $\mathbf{L}_F$ defining the filter (4) can be derived from the solutions of the matrix inequalities (23), (52), and (53) in the form

$$\mathbf{A}_F = \hat{\mathbf{A}}_F \hat{\mathbf{V}}_3^{-T}, \quad \mathbf{B}_F = \hat{\mathbf{B}}_F, \quad \mathbf{L}_F = \hat{\mathbf{L}}_F \hat{\mathbf{V}}_3^{-T}. \tag{55}$$

Proof: For a given matrix $\hat{\mathbf{V}}$, a matrix $\mathbf{V}$ satisfying (48) is

$$V = \begin{bmatrix} \hat{\mathbf{V}}_1 & \hat{\mathbf{V}}_2 \hat{\mathbf{V}}_3^{-T} \\ I & \hat{\mathbf{V}}_3^{-T} \end{bmatrix}. \tag{56}$$

Therefore, (55) follows by inversely deriving $\mathbf{A}_F$, $\mathbf{B}_F$, $\mathbf{L}_F$ from $\hat{\mathbf{A}}_F$, $\hat{\mathbf{B}}_F$, $\hat{\mathbf{L}}_F$ in (48) and (54). □

B. $\mathcal{H}_\infty$ Filter

The LMI-based formulation for the robust $\mathcal{H}_\infty$ filter can be obtained by a similar sequence of arguments.

- Choose the parameter-dependent Lyapunov variable as

$$\mathbf{Y}(\alpha) = \sum_{j=1}^{s} \alpha_j \mathbf{Y}_j. \tag{57}$$

- Partition (46) and the auxiliary variables $\hat{\mathbf{V}}$, $\hat{\mathbf{K}}$, $\hat{\mathbf{L}}$ defined from (48), (54), and

$$\hat{\mathbf{Y}}_j = \Pi_{\mathbf{V}}^T \mathbf{Y}_j \Pi_{\mathbf{V}}, \qquad j = 1, 2, \ldots, s$$

  with $\Pi_{\mathbf{V}}$ defined by (47).
- Apply the congruence transformation

$$\mathrm{diag}\,[\,\Pi_{\mathbf{V}} \quad \Pi_{\mathbf{V}} \quad I \quad I \quad I \quad \Pi_{\mathbf{V}}\,]$$

  to (41) in combination with the relations in (51).

It follows that the nonlinear matrix (41) reduces to the inequalities in (58), shown at the bottom of the page.

Theorem 5: There is a filter (4) that satisfies the robust estimation condition (39) whenever there is $\mu > 0$ such that the LMIs (24), (58) are feasible in $\hat{\mathbf{V}}$, $\hat{\mathbf{Y}}_j$, $\mathbf{S}_j$, $\mathbf{R}_j$, $\hat{\mathbf{K}}$, $\hat{\mathbf{L}}_F$, $\mathbf{G}$, and $\mathbf{F}$.

The filter data $\mathbf{A}_F$, $\mathbf{B}_F$, $\mathbf{L}_F$ defining the filter (4) can be derived from the solutions of the LMIs (24) and (58) according to the formulas in (55).

$$\begin{bmatrix} -\mu(\hat{\mathbf{V}} + \hat{\mathbf{V}}^T) & * & * & * & * \\ \Theta A_j^T \Theta^T \hat{\mathbf{V}} + \mathcal{C}_j^T \hat{\mathbf{K}}^T \mathcal{I} + \hat{\mathbf{X}}_j & -\hat{\mathbf{X}}_j & * & * & * \\ \mathcal{B}_j^T \Theta^T \hat{\mathbf{V}} + \mathcal{D}_j^T \Theta_p^T \hat{\mathbf{K}}^T \mathcal{I} & 0 & \begin{bmatrix} \mathbf{S}_{1j} & * \\ 0 & -I_m \end{bmatrix} & * & * \\ 0 & \mathbf{F}_1 C_{\Delta j}\Theta^T & \mathbf{F}_1 \mathcal{D}_{zj} & \mathbf{R}_{1j} - (\mathbf{F}_1 + \mathbf{F}_1^T) & * \\ \mu\hat{\mathbf{V}} & 0 & 0 & 0 & -\hat{\mathbf{X}}_j \end{bmatrix} < 0, \qquad j = 1, 2 \ldots, s \tag{52}$$

$$\begin{bmatrix} -\mu(\hat{\mathbf{V}} + \hat{\mathbf{V}}^T) & * & * & * & * & * \\ \Theta A_j^T \Theta^T \hat{\mathbf{V}} + \mathcal{C}_j^T \hat{\mathbf{K}}^T \mathcal{I} + \hat{\mathbf{Y}}_j & -\hat{\mathbf{Y}}_j & * & * & * & * \\ \mathcal{B}_j^T \Theta^T \hat{\mathbf{V}} + \mathcal{D}_j^T \Theta_p^T \hat{\mathbf{K}}^T \mathcal{I} & 0 & \begin{bmatrix} \mathbf{S}_j & * \\ 0 & -\gamma I_m \end{bmatrix} & * & * & * \\ 0 & \mathbf{F} C_{\Delta j}\Theta^T & \mathbf{F}\mathcal{D}_{zj} & \mathbf{R}_j - (\mathbf{F} + \mathbf{F}^T) & * & * \\ 0 & [\,L_j \quad -\hat{\mathbf{L}}_F\,] & [\,D_{\Delta\Delta j} \quad M_j\,] & 0 & -\gamma I_q & * \\ \mu\hat{\mathbf{V}} & 0 & 0 & 0 & 0 & -\hat{\mathbf{Y}}_j \end{bmatrix} < 0$$
$$j = 1, 2 \ldots, s \tag{58}$$

C. Mixed $\mathcal{H}_2/\mathcal{H}_\infty$ Filter

As the direct consequence of Theorems 4 and 5, we have the following result regarding the optimal mixed filter problem (5).

Theorem 6: Under the assumption (25), a suboptimal robust filter (4) for problem (5) can be solved by the following optimization problem:

$$\min_{\hat{\mathbf{V}},\hat{\mathbf{X}}_j,\hat{\mathbf{Y}}_j,\mathbf{S}_{ij},\mathbf{R}_{ij},\mathbf{S}_j,\mathbf{R}_j,\hat{\mathbf{K}},\hat{\mathbf{L}}_F,\mathbf{H}_i,\mathbf{F}_i,\mathbf{H},\mathbf{F},\mu,\nu,\gamma} \times[\rho\nu+(1-\rho)\gamma]: \text{(23), (24), (52), (53), and (58).} \quad (59)$$

The matrix data $\mathbf{A}_F$, $\mathbf{B}_F$, $\mathbf{L}_F$ defining the suboptimal filter (4) can be derived from the solutions of the optimization problem (59) according to the formulas in (55).

V. Numerical Examples

The example below clarifies how different model parameterizations as well as how different optimization formulations may lead to dramatically different filter performances. Consider the robust filtering for the system:

$$\begin{bmatrix}\dot{x}(t)\\ y(t)\\ z(t)\end{bmatrix}=\begin{bmatrix}A(\alpha) & B\\ C & D\\ L & 0\end{bmatrix}\begin{bmatrix}x(t)\\ w(t)\end{bmatrix} \quad (60)$$

with

$$\begin{aligned}
&A(\alpha)=Q_0+\alpha_1^3Q_1+\alpha_2^3Q_2+\alpha_1\alpha_2^2Q_3+\alpha_1Q_4+\alpha_2Q_5\\
&Q_0=\begin{bmatrix}-0.7 & -1.0\\ 0.1 & -0.5\end{bmatrix},\quad Q_1=\begin{bmatrix}0.3 & 0.2\\ 0.1 & 0.2\end{bmatrix}\\
&Q_2=\begin{bmatrix}0.2 & 0.1\\ 0.2 & 0.3\end{bmatrix},\quad Q_3=\begin{bmatrix}0.4 & 0.1\\ 0.15 & 0.1\end{bmatrix}\\
&Q_4=\begin{bmatrix}0.25 & 0.25\\ 0.1 & 0.25\end{bmatrix},\quad Q_5=\begin{bmatrix}0.25 & 0\\ 0.1 & 0.25\end{bmatrix}\\
&B=\begin{bmatrix}-2 & 0\\ 1 & 0\end{bmatrix},\quad C=[-100\quad 100]\\
&D=[0\quad 1],\quad L=[1\quad 0].
\end{aligned} \quad (61)$$

Two alternative representations of the uncertain system can be used in the construction of the filter.

- NFT

$$A(\alpha)=Q_0+\alpha_1Q_4+\alpha_2Q_5+[\alpha_1I_2\quad \alpha_2I_2]\begin{bmatrix}Q_1 & Q_3\\ 0_2 & Q_2\end{bmatrix}\times\begin{bmatrix}\alpha_1I_2 & 0_2\\ 0_2 & \alpha_2I_2\end{bmatrix}\begin{bmatrix}\alpha_1I_2\\ \alpha_2I_2\end{bmatrix} \quad (62)$$

which leads to NFT (1) with

$$\begin{aligned}
&A(\alpha)=\alpha_1(Q_0+Q_4)+\alpha_2(Q_0+Q_5)\\
&B_\Delta(\alpha)=[\alpha_1I_2\quad \alpha_2I_2]\begin{bmatrix}Q_1 & Q_3\\ O & Q_2\end{bmatrix}\\
&\Delta(\alpha)=\begin{bmatrix}\alpha_1I_2 & 0_2\\ 0_2 & \alpha_2I_2\end{bmatrix},\quad D_{\Delta z}=0\\
&C_\Delta(\alpha)=\begin{bmatrix}\alpha_1I_2\\ \alpha_2I_2\end{bmatrix},\quad D_z=0\\
&D_\Delta=0,\quad D_{\Delta\Delta}=0.
\end{aligned} \quad (63)$$

- LFT

$$\begin{aligned}
A(\alpha)&=Q_0+[I_2\ \ 0_2\ \ 0_2\ \ I_2\ \ 0_2\ \ 0_2]\,\Delta(\alpha)\\
&\times\left(I_{12}-\begin{bmatrix}0_2 & Q_1 & 0_2 & 0_2 & Q_3 & 0_2\\ 0_2 & 0_2 & I_2 & 0_2 & 0_2 & 0_2\\ 0_2 & 0_2 & 0_2 & 0_2 & 0_2 & 0_2\\ 0_2 & 0_2 & 0_2 & 0_2 & Q_2 & 0_2\\ 0_2 & 0_2 & 0_2 & 0_2 & 0_2 & I_2\\ 0_2 & 0_2 & 0_2 & 0_2 & 0_2 & 0_2\end{bmatrix}\Delta(\alpha)\right)^{-1}\\
&\times\begin{bmatrix}Q_4\\ 0_2\\ I_2\\ Q_5\\ 0_2\\ I_2\end{bmatrix},\quad \Delta(\alpha)=\begin{bmatrix}\alpha_1I_6 & O_6\\ O_6 & \alpha_2I_6\end{bmatrix}
\end{aligned} \quad (64)$$

which leads to the LFT in (1) with

$$\begin{aligned}
&A=Q_0,\quad B_\Delta=[I_2\quad 0_2\quad 0_2\quad I_2\quad 0_2\quad 0_2]\\
&D_{\Delta z}=\begin{bmatrix}0_2 & Q_1 & 0_2 & 0_2 & Q_3 & 0_2\\ 0_2 & 0_2 & I_2 & 0_2 & 0_2 & 0_2\\ 0_2 & 0_2 & 0_2 & 0_2 & 0_2 & 0_2\\ 0_2 & 0_2 & 0_2 & 0_2 & Q_2 & 0_2\\ 0_2 & 0_2 & 0_2 & 0_2 & 0_2 & I_2\\ 0_2 & 0_2 & 0_2 & 0_2 & 0_2 & 0_2\end{bmatrix},\quad C_\Delta=\begin{bmatrix}Q_4\\ 0_2\\ I_2\\ Q_5\\ 0_2\\ I_2\end{bmatrix}\\
&D_\Delta=0,\quad D_{\Delta\Delta}=0.
\end{aligned} \quad (65)$$

Note that the dimension 12 of z_Δ in the LFT (64) is three times larger than the one of the NFT in (62). This has a very detrimental effect on the computational efficiency and on the estimation performance of the filter, as described in Table I. Note that computations were performed with the MATLAB LMI control toolbox [7]. Note also that an averaged running time of LMI programs for NFT (62) is about 12 s, whereas its counterpart for LFT (64) is much longer. The tradeoff between the $\mathcal{H}_\infty$ and $\mathcal{H}_2$ performances by using both parameter-dependent Lyapunov function and single Lyapunov function are clearly indicated in Table II. The benefit obtained from the use of parameter-dependent Lyapunov functions is also significant in our computations.

Appendix
Proof of Theorem 1

Equation (9) ⇔ (10): Rewrite (10) as

$$\begin{bmatrix}0 & 0 & C^T & \mathbf{X}\\ 0 & Q_{11} & Q_{12} & 0\\ C & Q_{12}^T & Q_{22} & 0\\ \mathbf{X} & 0 & 0 & 0\end{bmatrix}+\left\{\begin{bmatrix}A^T\\ B^T\\ 0\\ -I\end{bmatrix}\mathbf{V}[I\quad 0\quad 0\quad \mu I]+(*)\right\}<0. \quad (66)$$

In order to use the projection lemma, we need to compute the

TABLE I
COMPUTATIONAL PERFORMANCES OF DIFFERENT MODEL REPRESENTATIONS

model	$\mathcal{H}_\infty$-performance	$\mathcal{H}_2$-performance	mixed performance for $\rho = 0.8$
NFT (62)	3.2592	0.0325	3.7545
LFT (64)	3.4590	3.5650	$+\infty$

TABLE II
PERFORMANCES OF FILTERS FOR NFT SYSTEMS WITH DIFFERENT WEIGHTS ρ AND USING PARAMETER-DEPENDENT LYAPUNOV FUNCTIONS OR WITH FIXED LYAPUNOV FUNCTIONS (IN PARENTHESIS)

ρ	mixed performance	$\mathcal{H}_\infty$ value	$\mathcal{H}_2$ value
0.1	6.7389(7.6133)	3.7082(14.4215)	7.0756(6.8569)
0.3	6.0244(8.4078)	3.3862(9.2827)	7.1551(8.0328)
0.4	5.6376(8.4329)	3.2381(8.0423)	7.2373(8.6934)
0.5	5.2236(8.2874)	3.0869(7.1583)	7.3603(9.4164)
0.7	4.2965(7.4454)	2.7696 (5.5167)	7.8593(11.9458)
0.9	3.1177(5.5604)	2.4107(4.2182)	9.4810(17.6399)

nullspaces

$$\mathcal{N}_{[I \;\; 0 \;\; 0 \;\; \mu I]} = \begin{bmatrix} -\mu I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \\ I & 0 & 0 \end{bmatrix}$$

$$\mathcal{N}_{[A \;\; B \;\; 0 \;\; -I]} = \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \\ A & B & 0 \end{bmatrix}. \tag{67}$$

Thus, by the projection lemma, (66) is feasible in $\mathbf{V}$ if and only if

$$(*) \begin{bmatrix} 0 & 0 & C^T & \mathbf{X} \\ 0 & Q_{11} & Q_{12} & 0 \\ C & Q_{12}^T & Q_{22} & 0 \\ \mathbf{X} & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \\ A & B & 0 \end{bmatrix} = \begin{bmatrix} A^T\mathbf{X} + \mathbf{X}A & \mathbf{X}B & C^T \\ B^T\mathbf{X} & Q_{11} & Q_{12} \\ C & Q_{12}^T & Q_{22} \end{bmatrix} < 0 \tag{68}$$

and

$$(*) \begin{bmatrix} 0 & 0 & C^T & \mathbf{X} \\ 0 & Q_{11} & Q_{12} & 0 \\ C & Q_{12}^T & Q_{22} & 0 \\ \mathbf{X} & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} -\mu I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \\ I & 0 & 0 \end{bmatrix} = \begin{bmatrix} -2\mu\mathbf{X} & 0 & -\mu C^T \\ 0 & Q_{11} & Q_{12} \\ -\mu C & Q_{12}^T & Q_{22} \end{bmatrix} < 0$$

$$\Leftrightarrow \mathbf{X} - \mu/2 \, [0 \;\; C^T] \begin{bmatrix} Q_{11} & Q_{12} \\ Q_{12}^T & Q_{22} \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ C \end{bmatrix} < 0$$
(by Schur's complement). (69)

Inequality (66) is therefore equivalent to (68) and (69), which readily imply (9). Conversely, assume we know a solution to (9). It is also a solution to (68) and (69), provided that μ is chosen to be a sufficiently small positive quantity. This proves that (9) implies (10) by virtue of the projection lemma.

Equation (11) $\Leftrightarrow$ (9): If (9) is feasible in $\mathbf{X}$, then (11) can be readily shown to be feasible for $\mathbf{V} = \mathbf{X}$, $\mathbf{V}_1 = \mu\mathbf{X}$ with μsufficiently small.

Conversely, suppose there are $\mathbf{X}$, $\mathbf{V}$, $\mathbf{V}_1$ satisfying (11). This can be rewritten as

$$\begin{bmatrix} 0 & * & * & * \\ 0 & Q_{11} & * & * \\ C & Q_{12}^T & Q_{22} & * \\ \mathbf{X} & 0 & 0 & 0 \end{bmatrix} + \left\{ \begin{bmatrix} A^T \\ B^T \\ 0 \\ -I \end{bmatrix} [\mathbf{V} \;\; 0 \;\; 0 \;\; \mathbf{V}_1] + (*) \right\} < 0. \tag{70}$$

Again, we obtain (9) by projecting onto $\mathcal{N}_{[A \;\; B \;\; 0 \;\; -I]}$.

Equation (12) $\Rightarrow$ (9): Rewrite (12) as

$$\begin{bmatrix} 0 & \mathbf{X} & 0 & 0 & 0 \\ \mathbf{X} & -\mu\mathbf{X} & 0 & 0 & 0 \\ 0 & 0 & Q_{11} & Q_{12} & 0 \\ 0 & C & Q_{12}^T & Q_{22} & 0 \\ 0 & 0 & 0 & 0 & -\mathbf{X}/\mu \end{bmatrix} + \left\{ \begin{bmatrix} -I \\ A^T \\ B^T \\ 0 \\ I \end{bmatrix} \mathbf{V} [I \;\; 0 \;\; 0 \;\; 0 \;\; 0] + (*) \right\} < 0. \tag{71}$$

Again, the explicit form of relevant nullspaces are

$$\mathcal{N}_{[-I \;\; A \;\; B \;\; 0 \;\; I]} = \begin{bmatrix} A & B & 0 & I \\ I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & I \end{bmatrix}$$

$$\mathcal{N}_{[I \;\; 0 \;\; 0 \;\; 0 \;\; 0]} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & I \end{bmatrix}.$$

From the projection lemma applied to (71) with respect to $\mathbf{V}$, we infer that

$$(*)\begin{bmatrix} 0 & \mathbf{X} & 0 & 0 & 0 \\ \mathbf{X} & -\mu\mathbf{X} & 0 & C^T & 0 \\ 0 & 0 & Q_{11} & Q_{12} & 0 \\ 0 & C & Q_{12}^T & Q_{22} & 0 \\ 0 & 0 & 0 & 0 & -\mathbf{X}/\mu \end{bmatrix} \begin{bmatrix} A & B & 0 & I \\ I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & I \end{bmatrix} = \begin{bmatrix} A^T\mathbf{X}+\mathbf{X}A-\mu\mathbf{X} & \mathbf{X}B & C^T & \mathbf{X} \\ B^T\mathbf{X} & Q_{11} & Q_{12} & 0 \\ C & Q_{12}^T & Q_{22} & 0 \\ \mathbf{X} & 0 & 0 & -\mathbf{X}/\mu \end{bmatrix} < 0 \quad (72)$$

and

$$(*)\begin{bmatrix} 0 & \mathbf{X} & 0 & 0 & 0 \\ \mathbf{X} & -\mu\mathbf{X} & 0 & C^T & 0 \\ 0 & 0 & Q_{11} & Q_{12} & 0 \\ 0 & C & Q_{12}^T & Q_{22} & 0 \\ 0 & 0 & 0 & 0 & -\mathbf{X}/\mu \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 \\ I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & I \end{bmatrix} = \begin{bmatrix} -\mu\mathbf{X} & 0 & C^T & 0 \\ 0 & Q_{11} & Q_{12} & 0 \\ C & Q_{12}^T & Q_{22} & 0 \\ 0 & 0 & 0 & -\mathbf{X}/\mu \end{bmatrix} < 0. \quad (73)$$

Now, (72) is trivially equivalent to (9) by a Schur's complement argument.

Equation (9) $\Rightarrow$ (12): Clearly, for $\mathbf{X}$ satisfying (9), there is a $\mu > 0$ such that (73) holds. By the projection lemma, this, together with (72) [which is equivalent to (9)], is sufficient for the existence of $\mathbf{V}$ satisfying (12). Furthermore, for $C = 0$, (73) automatically holds true for $\mu = 1$, and thus, together with (72), implies (14) as well.

Finally, the equivalence between (12) and (13) follows from the congruence transformation diag$[\,\mu I \quad I \quad I \quad I \quad \mu I\,]$ applied to (12) and the change of variables $\mathbf{X} \leftarrow \mu\mathbf{X}$, $\mathbf{V} \leftarrow \mu\mathbf{V}$. □

REFERENCES

[1] B. D. O. Anderson and J. B. Moore, *Optimal Filtering*. Englewood Cliffs, NJ: Prentice-Hall, 1979.

[2] P. Apkarian and H. D. Tuan, "Parameterized LMI's in control theory," *SIAM J. Contr. Optim.*, vol. 38, pp. 1241–1264, 2000.

[3] P. Apkarian, H. D. Tuan, and J. Bernussou, "Continuous-time analysis, eigenstructure assignment and $\mathcal{H}_2$ synthesis with enhanced LMI characterizations," in *Proc. 39th CDC*, Sydney, Australia, 2000, pp. 1489–1494. see also *IEEE Trans. Automat. Contr.*, vol. 46, pp. 1941–1946, Sept. 2001.

[4] P. Apkarian, P. Pellanda, and H. D. Tuan, "Mixed $\mathcal{H}_2/\mathcal{H}_\infty$ multi-channel linear parameter-varying control in discrete time," *Syst. Contr. Lett.*, vol. 41, pp. 333–346, 2000.

[5] C. K. Chui and G. Chen, *Kalman Filtering*, Third ed. New York: Springer, 1999.

[6] P. Gahinet and P. Apkarian, "A linear matrix inequality approach to $\mathcal{H}_\infty$ control," *Int. J. Nonlinear Robust Contr.*, no. 4, pp. 421–448, 1994.

[7] P. Gahinet, A. Nemirovski, A. Laub, and M. Chilali, *LMI Control Toolbox*. Natick, MA: MathWorks, 1995.

[8] J. C. Geromel, "Optimal linear filtering under parameter uncertainty," *IEEE Trans. Signal Processing*, vol. 47, pp. 168–175, Jan. 1999.

[9] P. Khargonekar, M. Rotea, and E. Baeyens, "Mixed $\mathcal{H}_2/\mathcal{H}_\infty$ filtering," *Int. J. Nonlinear Robust Contr.*, vol. 6, pp. 313–330, 1996.

[10] H. Li and M. Fu, "A linear matrix inequality approach to robust $\mathcal{H}_\infty$ filtering," *IEEE Trans. Signal Processing*, vol. 45, pp. 2338–2350, Sept. 1997.

[11] M. C. de Oliveira, J. Bernussou, and J. C. Geromel, "A new discrete-time robust stability conditions," *Syst. Contr. Lett.*, vol. 37, pp. 261–265, 1999.

[12] H. Rotstein, M. Sznaier, and M. Idan, "$\mathbf{H}_1/\mathcal{H}_\infty$ filtering theory and an aerospace application," *Int. J. Nonlinear Robust Contr.*, vol. 6, pp. 347–366, 1996.

[13] C. Scherer, "A full block $\mathcal{S}$-procedure with applications," in *Proc. IEEE Conf. Decision Contr.*, San Diego, CA, 1997, pp. 2602–2607.

[14] C. E. de Souza and A. Trofino, "An LMI approach to the design of robust $\mathcal{H}_2$ filters," in *Recent Advances on Linear Matrix Inequality Methods in Control*, L. El Ghaoui and S. Niculescu, Eds. Philadelphia, PA: SIAM, 1999.

[15] Y. Theodor and U. Shaked, "A dynamic game approach to mixed $\mathcal{H}_\infty/\mathcal{H}_2$ estimation," *Int. J. Nonlinear Robust Contr.*, vol. 6, pp. 331–345, 1996.

[16] H. D. Tuan and P. Apkarian, "Relaxations of parameterized LMI's with control applications," *Int. J. Nonlinear Robust Contr.*, vol. 9, pp. 59–84, 1999.

[17] H. D. Tuan, P. Apkarian, and T. Q. Nguyen, "Robust and reduced-order filtering: New characterizations and methods," in *Proc. Amer. Contr. Conf.*, 2000, pp. 1327–1331. see also *IEEE Trans. Signal Processing*, vol. 49, pp. 2975–2984, Dec. 2001.

[18] K. Zhou, J. C. Doyle, and K. Glover, *Robust and Optimal Control*. Englewood Cliffs, NJ: Prentice-Hall, 1996.

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