Robust Control Approaches for a Two-Link Flexible Manipulator

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Abstract

Control approaches for a two-link flexible manipulator are studied in the context of robust synthesis for Linear Parameter-Varying systems. Different treatments of the inertia matrix variations in the manipulator system are examined in three control law design formulations. The first two designs are based upon scaled H_∞ or structured singular value synthesis. The third design makes use of a new approach for robust gain-scheduled synthesis. Results show that this gain-scheduling technique maximizes both performance and robustness over the entire range of manipulator configurations.

1 Introduction

Flexible manipulators can take many forms and serve many purposes. The best known flexible manipulator is probably the space shuttle's remote manipulator system (RMS), but there are many others which are much closer to home. Many devices that one sees every day, such as a construction crane or a cherry picker, can also be examples. The study of closed-loop control for these types of devices has been motivated primarily by the combined need for lightweight structures, large work spaces, precision tracking, and disturbance rejection. The first requirement is driven by cost, the latter three by system level performance specifications. It is in space systems where these requirements most often lead to a challenging closed-loop control problem.

While of little use in practical applications, a two-link flexible manipulator is an excellent benchmark for the study of the control issues for flexible systems. From a feedback control viewpoint, it brings with it the critical problems of flexibility, modeling uncertainty, and varying inertial properties. It remains a truly nonlinear problem and thus resists purely linear approaches to control law design. A number of different nonlinear techniques have been proposed for the calculation of control laws: a passive design approach is presented by Juang *et al.*[1], Madhavan & Singh[2] and Khorrami *et al.*[3] suggest feedback linearization combined with an outer loop linear controller, Su & Leung[4] derive sliding mode control laws under rigid body assumptions. Finally, an observer based H_∞ approach is used by Meressi & Paden[5] to derive a gain-scheduled controller for the two-link flexible manipulator control problem.

In this paper, three different control law design approaches for flexible manipulators are examined. The goal is to highlight the critical issues and trade-offs in control synthesis for these systems and then to present techniques for dealing with them. The work is limited to the framework of robust synthesis for Linear Parameter-Varying systems. It is left to the reader to compare these approaches to alternative techniques such as those mentioned above.

This paper is organized as follows. In Section 2, some notations and definitions are presented to provide a context for the design formulations which follow. Section 3 describes the two-link flexible manipulator control problem and the laboratory experiment SECAFLEX. The design formulations for the three different approaches are presented in Section 4. First, the common elements of the synthesis problem are described. The three control synthesis formulations are then presented, progressing from the simplest to the most advanced approach. Section 5 presents and compares the results for the different techniques by examining both the controllers and the resulting linear closed-loop time responses. Section 6 finishes with conclusions and final observations.

2 Notation

The goal of this section is to provide the required nomenclature and a brief overview of the underlying theory used in this work. The notation $\mathbf{diag}(X_1, \ldots, X_N)$ is used to represent the block-diagonal matrix

$$\begin{bmatrix} X_1 & 0 & \cdots & 0 \\ 0 & X_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & X_N \end{bmatrix}.$$

Linear Fractional Transformations or LFTs will be used to describe system interconnections. For matrices K and $P = \begin{pmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{pmatrix}$ of appropriate dimension and assuming that $(I - P_{22}K)$ has full rank, the lower LFT is defined as

$$F_l(P, K) = P_{11} + P_{12}K(I - P_{22}K)^{-1}P_{21}. \tag{1}$$

and assuming that $(I - P_{11}K)$ has full rank, the upper LFT is

$$F_u(P, K) = P_{22} + P_{21}K(I - P_{11}K)^{-1}P_{12}. \tag{2}$$

The term Linear Parameter-Varying (LPV) system refers to a system of the form,

$$\dot{x} = A(\theta)x + B(\theta)w \tag{3}$$

$$z = C(\theta)x + D(\theta)w, \tag{4}$$

where $\theta(t)$ is a vector of time-varying parameters. The notation $\sigma(A)$ represents the singular values of matrix A.

For a fixed parameter vector θ, the system in eqns. (3) and (4) becomes Linear Time-Invariant (LTI). A design technique that targets robust performance for LTI systems is μ-synthesis. The synthesis procedure is driven by the so called $D - K$ iteration scheme(in our case $S - K$ iteration) which consists of sequentially designing controllers and scaling functions which minimize an upper bound on the scaled H_∞ norm of the closed-loop system (Doyle[6], Safonov[7], and Doyle[8]). The controller design step is performed using H_∞ synthesis. The scaling step can be handled by a number of different algorithms, depending on the uncertainty structure. The iterative procedure does not have guaranteed global convergence, but has been shown to work well in many applications. The μ-synthesis procedure is used in the first two designs presented in this paper.

An approach similar to μ-synthesis, but for LPV systems has been developed using the Bounded Real Lemma (Anderson[9]) and Parameter-Dependent Lyapunov functions (Wu *et al.*[10], Apkarian & Adams[11]). This technique, which targets LPV robust performance, is used in the last design in this paper.

3 Problem Description

The control of a two-link flexible manipulator is a problem with a number of simultaneous and sometimes conflicting requirements. First of all, both rigid body and lightly damped structural modes must be stabilized. The problem is complicated by uncertainty in the high frequency dynamics of the system. This is due to the unreliability of modeling and system identification approaches above some limited bandwidth. Next, the maximum possible performance should be achieved, measured by settling time, overshoot, and disturbance rejection. Finally, the variation in the system's inertial properties as a function of manipulator geometry must be considered. This last requirement is potentially the most challenging, since in the strictest sense it is a truly nonlinear control problem.

The laboratory structure SECAFLEX is used in this work as a benchmark for the study of flexible manipulator control problems. It is a two link flexible planar manipulator driven by geared DC motors, built as a laboratory platform for control-structure interaction experiments at CERT-ONERA in Toulouse, France. Its two flexible members are homogeneous beams. There is a concentrated mass at the elbow due to the DC motor and a concentrated mass at the tip of the second beam which is the payload. The modeling of the manipulator has been studied extensively. Approaches to formulating the equations of motion include an analytic form based on Euler-Bernoulli cantilever-free modes, the introduction of fictitious springs, and a finite-element based technique (Alazard & Chrétien[12]). A simplified drawing of this two-link manipulator is shown in Fig. 1. θ_1 and θ_2 are respectively the shoulder and elbow joint angles. τ_1 and τ_2 are the corresponding control torques. The second-order form of the manipulator equations of

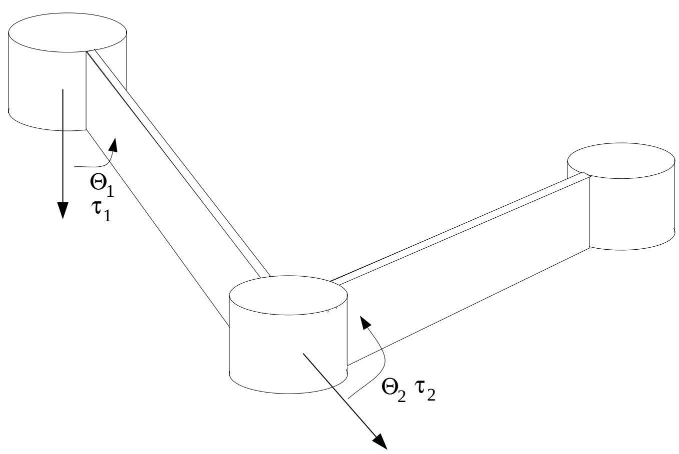

Figure 1: Two-Link Flexible Manipulator

motion can be written as

$$M(\theta_2)\ddot{q}(t) + D\dot{q}(t) + Kq(t) = Fr(t) + C(q(t), \dot{q}(t)), \tag{5}$$

where M, D, K, and F are the inertia, damping, stiffness, and control distribution matrices respectively. $C(q(t), \dot{q}(t))$ is a vector of nonlinearities due to Coriolis and centripetal forces. $r(t)$ is the input vector, $r = [\ \tau_1\ \tau_2\ ]^T$. The input vector can be redefined to approximately cancel out the nonlinear forces

$$r(t) = u(t) - C'(q(t), \dot{q}(t)) \tag{6}$$

to give us the second-order LPV form of the equations of motion,

$$M(\theta_2)\ddot{q}(t) + D\dot{q}(t) + Kq(t) = Fu(t) \tag{7}$$

Due to the variable geometry of the system, the inertia matrix is a function of the second joint angle, θ_2. This dependence causes significant changes in the response of the system to input torques over the range of possible configurations. If we consider an output vector,

$$y = [\ \theta_1\ \theta_2\ ]^T = Hq(t), \tag{8}$$

then we can define the family of linearized transfer functions from u to y as $G(s, \bar{\theta}_2)$. The parameter $\bar{\theta}_2$ denotes a fixed value of θ_2 at which a linearization is performed. Fig. 2 illustrates the variation of the manipulator dynamics with geometry by showing the singular values of $G(s, \bar{\theta}_2)$ at three different values of θ_2. The values θ_2 in $[0, \pi]$ capture the range of all possible inertia matrix variations. Symmetry provides that $M(\theta_2) = M(2\pi - \theta_2)$ for all θ_2 in $[\pi, 2\pi]$.

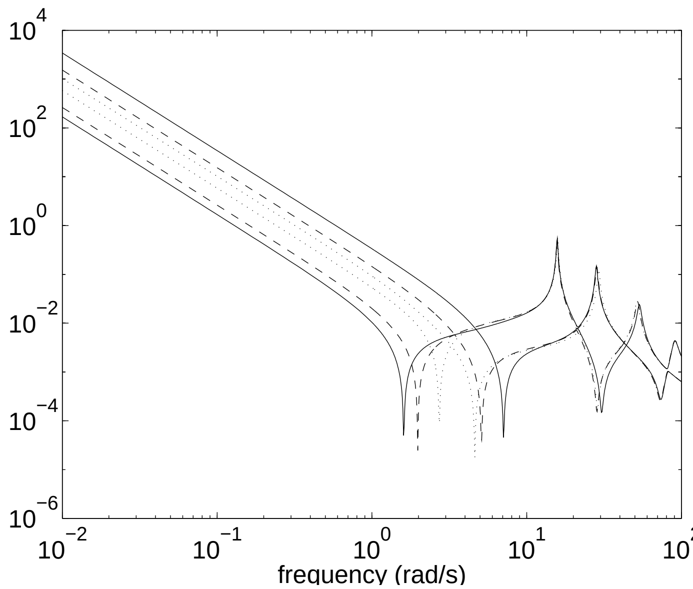

Figure 2: $\sigma(G(j\omega, \bar{\theta}_2))$
$\theta_2 = 0$ (solid), $\pi/2$ (dashed), π (dotted)

4 Control Problem Formulation

The general framework of H_∞ and the structured singular value is used in this study for its ability to handle multi-specification problems. Within this framework, the performance objective of command following can be handled by a weighed minimization of the sensitivity function. A frequency dependent weight, W_p, is formulated that penalizes the error, e, between angular position commands, w_1, and the output, y. By forcing this weighted sensitivity function to be small, the complementary sensitivity function approaches identity at low frequencies, thus providing good command tracking.

The performance objective for the two-link manipulator is a rapid and decoupled response to position commands with minimal overshoot. The achievable performance is greatly influenced by the formulation of robustness requirements and gain-scheduling. With this considered, we find that the performance weighting function becomes an instrument for tuning the design. It is used to achieve the maximum robust performance, that is the best performance in the presence of all uncertainties permitted by the synthesis model. We restrict the selection to first order weights of the form,

$$W_p(s) = \frac{a(s+b)}{s+c} I_{2\times 2} \tag{9}$$

The requirement for robustness to high frequency unmodeled dynamics can be included in the synthesis model as an additive uncertainty model. An additive uncertainty weight can be formulated by considering the difference between some high order geometry dependent model, $G(s,\bar{\theta}_2)$, and some reduced order design model, $G_r(s,\bar{\theta}_2)$. The design model is of lower order but still dependent on manipulator geometry.

Let W_f be an additive weight and Δ_f a complex uncertainty block, scaled such that $\sigma_{max}(\Delta_f) < 1$. The error between the full-order and reduced-order models is defined as $E(s,\bar{\theta}_2)$,

$$E(s,\bar{\theta}_2) = G(s,\bar{\theta}_2) - G_r(s,\bar{\theta}_2) \tag{10}$$

The additive uncertainty weight must then provide a frequency domain bound of this error, that is,

$$|W_f(j\omega)| \geq \max_{\theta_2} \sigma_{max}\{E(j\omega,\bar{\theta}_2)\} \; for \; all \; \omega \in [0,\infty) \tag{11}$$

For the SECAFLEX application, this weight has been defined as,

$$W_f(s) = \frac{4(s+.1)^2}{(s+100)^2} I_{2\times 2}. \tag{12}$$

The second-order system matrices which result from the fictitious springs modeling approach, including one flexible mode for each beam, follow. This is the manipulator model used for control law synthesis in this study. The dependence of the inertia matrix on θ_2 can be expressed as,

$$M(\theta_2) = M(\pi/2) + cos(\theta_2)(M(\pi/2) - M(\pi)) \tag{13}$$

where

$$M(\pi/2) = \begin{bmatrix} 34.7077 & 9.7246 & 23.6398 & 5.9114 \\ 9.7246 & 9.8783 & 9.7246 & 5.9114 \\ 23.6398 & 9.7246 & 17.5711 & 5.9114 \\ 5.9114 & 5.9114 & 5.9114 & 3.7233 \end{bmatrix}$$

and

$$M(\pi) = \begin{bmatrix} 17.0296 & 0.8856 & 9.7776 & 0.8430 \\ 0.8856 & 9.8783 & 4.7016 & 5.9114 \\ 9.7776 & 4.7016 & 7.5249 & 3.0311 \\ 0.8430 & 5.9114 & 3.0311 & 3.7233 \end{bmatrix}$$

The damping, stiffness, control effectiveness, and output matrices are respectively,

$$D = \mathbf{diag}(0, 0, 0.09, 0.05)$$

$$K = \mathbf{diag}(0, 0, 89.1473, 45.6434)$$

$$F = \begin{bmatrix} I_{2\times 2} \\ 0_{2\times 2} \end{bmatrix}$$

$$H = [I_{2\times 2} \;\; 0_{2\times 2}]$$

The manipulator equations of motion can be rewritten in first order form as,

$$\dot{x}(t) = \begin{bmatrix} 0 & I \\ -M(\theta_2)^{-1}K & -M(\theta_2)^{-1}D \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ M(\theta_2)^{-1}F \end{bmatrix} u(t) \qquad (14)$$

$$y = [\, H \;\; 0 \,]\, x(t) \qquad (15)$$

The last requirement, to handle the nonlinear nature of the manipulator dynamics motivated the development of three different approaches to control synthesis for this problem. Each design treats the inertia matrix variations differently in the design formulation. In Design #1, the variations are simply neglected in the synthesis model. In this case, a controller is found at some nominal condition and then analyzed over the set of admissible geometries. In Design #2, the inertia matrix variations are captured in a parametric uncertainty model. The design goal in this case is to find a single controller which gives robust performance for all $\theta_2 \in [0, \pi]$. Designs #1 and 2 are performed using a $D - K$ iteration procedure to minimize the scaled H_∞ norm of the closed-loop system. In design #3, the synthesis procedure described by Apkarian & Adams[11] is used to directly synthesize a parameter varying controller which guarantees both performance and robustness to high frequency unmodeled dynamics.

4.1 Design #1

For this first design, the simple approach is taken to synthesize the best possible controller at some nominal manipulator geometry, $\theta_2 = \pi/2$. All variations in the inertia matrix are neglected. This design provides a benchmark for those that follow since it shows the maximum performance that can be achieved with only performance and high frequency roll-off enforced. The synthesis model for this case is illustrated in Fig. 3. The performance

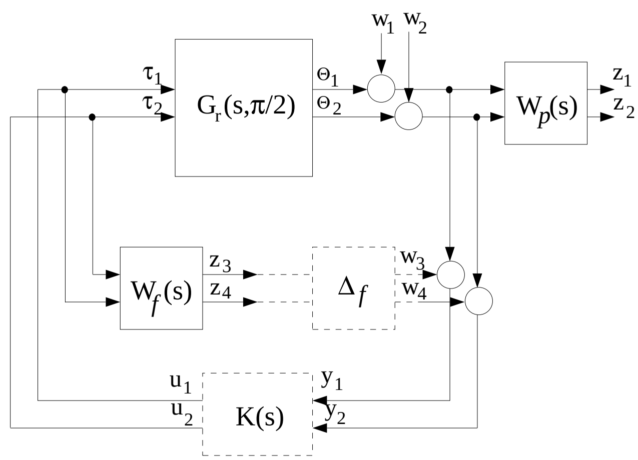

Figure 3: Synthesis Model for Design #1

weight used for this design is

$$W_p(s) = \frac{0.45(s+1)}{s+0.03} I_{2\times 2} \tag{16}$$

In the final $D-K$ iteration, the transfer function between the inputs $w_1, \ldots, w_4$ and outputs $z_1, \ldots, z_4$ is scaled by the matrix,

$$S = \mathbf{diag}(0.064 I_{2\times 2}, I_{2\times 2}) \tag{17}$$

4.2 Design #2

In the second design, the inertia matrix variations of the manipulator are treated as parametric uncertainty. The goal is thus to find a robust controller which guarantees performance in the presence of both high frequency unmodeled dynamics and changes in inertial properties. The approach is very conservative since an invariant controller is asked to provide robust performance for a system with time-varying parameters.

Consider the second order state space model of the system,

$$M(\theta_2)\ddot{q}(t) + D\dot{q}(t) + Kq(t) = Fu(t) \tag{18}$$

We would like to represent the variation in the inertia matrix in a non-conservative manner. A non-conservative linear fractional representation of these changes can be found if the variation in the inertia matrix can be written as an affine function. By letting $\rho = cos(\theta_2)$, the model can be rewritten as,

$$[M_0 + M_\rho \rho]\ddot{q}(t) + D\dot{q}(t) + Kq(t) = Fu(t) \tag{19}$$

A singular value decomposition can then be used to find matrices Γ_ρ and W_ρ such that $M_\rho = \Gamma_\rho W_\rho$. The column dimension of Γ_ρ and row dimension of W_ρ are equal to the rank, r, of the matrix M_ρ. The system can now be rewritten in descriptor form

$$[E + \Gamma(\rho I_{r\times r})W]\dot{x}(t) = Ax(t) + Bu(t) \tag{20}$$

where

$$E = \begin{bmatrix} I & 0 \\ 0 & M_0 \end{bmatrix},\ A = \begin{bmatrix} 0 & I \\ -K & -D \end{bmatrix},\ B = \begin{bmatrix} 0 \\ F \end{bmatrix}, \tag{21}$$

$$\Gamma = \begin{bmatrix} 0 \\ \Gamma_\rho \end{bmatrix},\ W = \begin{bmatrix} 0 & W_\rho \end{bmatrix} \tag{22}$$

An LFT description of the above system follows. Let us define a new system, $\tilde{G}_r(s) = \tilde{C}(sI - \tilde{A})^{-1}\tilde{B} + \tilde{D}$, such that $G_r(s, \bar{\theta}_2) = F_l(\tilde{G}_r(s), \rho I_{2\times 2})$. This new system is defined by the matrices (Adams & Chrétien[13]),

$$\tilde{A} = [E^{-1}A],\ \tilde{B} = [E^{-1}B \ \ -E^{-1}\Gamma], \tag{23}$$

$$\tilde{C} = \begin{bmatrix} C \\ WE^{-1}A \end{bmatrix},\ \tilde{D} = \begin{bmatrix} 0 & 0 \\ WE^{-1}B & -WE^{-1}\Gamma \end{bmatrix} \tag{24}$$

By capturing the inertia matrix variations with manipulator geometry as an LFT, the robust control problem can be formulated such that for all $\rho \in [-1, 1]$ and equivalently for all $\theta_2 \in [0, \pi]$ the closed-loop system will remain stable and achieve a certain degree of performance. Since this μ-synthesis solution must consider all uncertainties as complex, there is some conservatism built into the procedure.

The μ-synthesis design model is formed by combining the performance and robustness formulations into a single multi-specification synthesis model. This model is illustrated in Fig. 4. In the scaling step of the D-K iteration, the transfer function between the inputs $w_1, \ldots, w_6$ and outputs $z_1, \ldots, z_6$ is scaled by a proper transfer function matrix with the structure $S = \{\mathbf{diag}(s_1 I_{2\times 2} \ \ s_2 I_{2\times 2} \ \ S_1) : \ s_1, s_2 \in \mathbf{R},\ s_1, s_2 > 0,\ S_1 \in \mathbf{C}^{2\times 2},\ S_1 = S_1^* > 0\ \}$. Unfortunately, currently available commercial software do not allow for non-diagonal scaling blocks. In addition, the curve-fitting based approaches to scaling calculation offered by existing toolboxes can be unreliable for systems with lightly damped flexible modes. These shortcomings

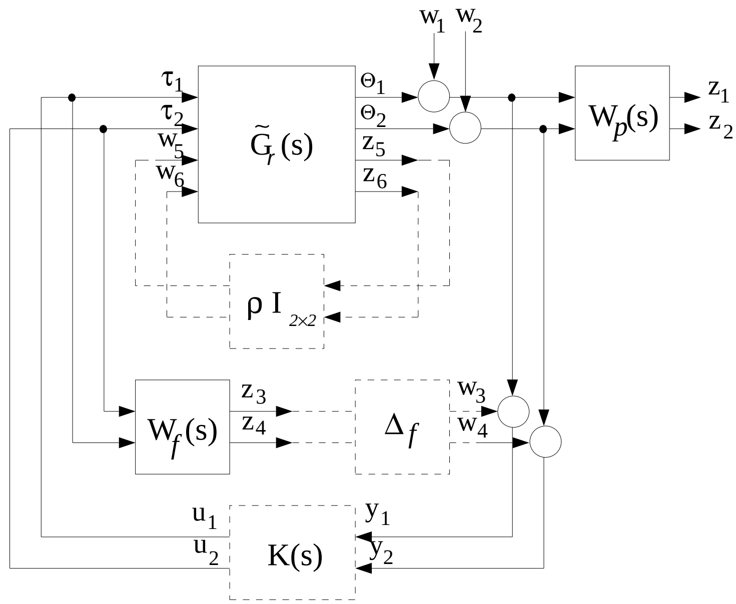

Figure 4: Synthesis Model for Design #2

led to the development of new algorithms for improved scaling function calculation as a part of this effort (Adams & Chrétien[14]).

The performance weighting function used for this design is,

$$W_p(s) = \frac{0.9(s+0.03)}{(s+1)} I_{2\times 2} \tag{25}$$

This somewhat unconventional high-pass weighting results from a difficulty in reducing overshoot in the closed-loop step responses. The uncertainty block between the input w and output z is now $\Delta = \mathbf{diag}(\Delta_{perf},\ \Delta_f,\ \rho I_{2\times 2})$. The final scaling function of the D-K iteration for this problem is static and given by,

$$S = \mathbf{diag}(0.153 I_{2\times 2}, I_{2\times 2}, \begin{bmatrix} 0.427 & -0.010 \\ -0.010 & 0.205 \end{bmatrix}). \tag{26}$$

4.3 Design #3

The previous design formulation relies on small gain theory (Desoer & Vidyasagar[15]) to guarantee robustness with respect to parameter variations in the inertia matrix of the flexible manipulator. This approach is conservative for three reasons: that ρ is treated as a complex number, that no restriction is placed on the rate of variation of ρ, and finally that we limited ourselves to linear time-invariant controllers. This conservatism was a strong motivation in the development of the gain-scheduling techniques based upon the concept of parameter-dependent Lyapunov functions described by Wu *et al.* [10], Feron *et al.* [16], and Apkarian *et al.* [17]. These techniques provide a systematic treatment of the gain-scheduling problem. Gain-scheduled controllers are characterized via Linear Matrix Inequalities (LMIs) which are readily solved using available convex semi-definite programming software (Gahinet *et al.* [18]). The reader is referred to Wu *et al.* [10], Becker*et al.* [19], and Apkarian & Adams[11] for more details on these techniques. The basic idea of these approaches is to exploit knowledge of the both the range of possible values of the parameter θ_2 and its rate of variation $\dot{\theta}_2$.

A scaling approach, analogous to that used in the $D - K$ iterations of our previous μ-synthesis designs, has been implemented to create a complete methodology for the gain-scheduled control of uncertain systems (Apkarian & Adams[11]). In addition to the real versus complex and rate of variation considerations, this technique further reduces conservatism in the design by taking advantage of parameter dependent scaling matrices.

For the two-link flexible manipulator problem, there is only one time-varying parameter, the second arm angle θ_2. As described previously, its value can be restricted by symmetry to $\theta_2 \in [0,\ \pi]$. A realistic limit on the rate of variation can be determined by considering the angular constraints of the manipulator and the maximum available control torques, $|\dot{\theta}_2| \leq 100\, deg/s$.

The design model which issues the plant data used in the LMI formulation is shown in Fig. 5. It is interesting to note the similarities between this

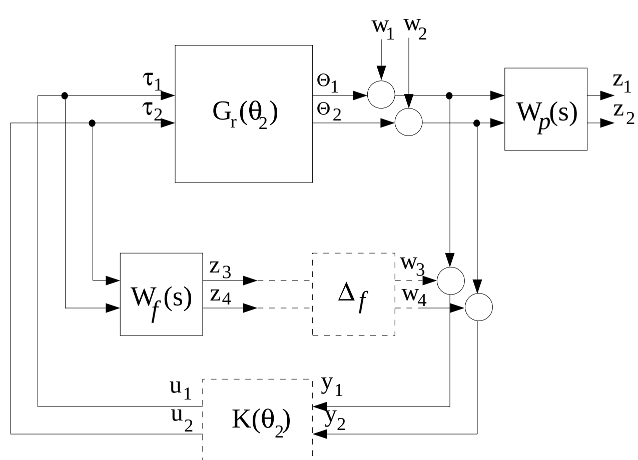

Figure 5: Synthesis Model for Design #3

design model and the one used in design #1, Fig. 3. In satisfying similar objectives, we are now solving the controller synthesis problem for all admissible operating conditions. It is also worthwhile to observe the contrasts between designs #2 and #3. Unlike design #2, there is no parameter uncertainty block in design #3 corresponding to the changing inertia matrix. These variations are no longer treated as uncertainty at all, but rather as a measurable output injected into the controller dynamics.

The performance weight used for direct gain-scheduled synthesis is,

$$W_p(s) = \frac{0.1(s+1)}{(s+0.03)} I_{2\times 2} \tag{27}$$

The uncertainty block between w and z is $\Delta = \mathbf{diag}(\Delta_{perf},\ \Delta_f)$. The final scaling function for this problem is,

$$S^{-1}(\theta_2) := \begin{bmatrix} 3.363 I_{2\times 2} & 0 \\ 0 & 0.020 I_{2\times 2} \end{bmatrix} + cos(\theta_2) \begin{bmatrix} 0.074 I_{2\times 2} & 0 \\ 0 & 2e^{-4} I_{2\times 2} \end{bmatrix} \tag{28}$$

It should be noted that the scheduling parameter in this design is not an externally evolving variable, uncorrelated with the system dynamics. The scheduling variable, θ_2, is also a state of the manipulator system. By treating its value as external to the system, we are ignoring the fact that the evolution of θ_2 is restricted not just to some hypercube of positions and rates, but by the nonlinear dynamics of the system. The result is therefore some degree of design conservatism. Obviously, this last design approach does the best job in capturing the physical reality of the problem. If, due to this conservatism, satisfactory closed-loop performance was still not achieved with this approach, one might be led to investigate nonlinear H_∞ techniques. To handle multiple design specifications, some form of nonlinear $D-K$ iteration would have to be derived. We will see in the next section that this is thankfully not necessary.

5 Results

This section presents the results of the three design formulations described above. For each design, we first look at the resulting controller's frequency response and how it varies (if at all) with manipulator geometry. Step responses are shown at three different values of θ_2 in order to illustrate how well the controller regulates the system over the entire range of parameters. The full order manipulator model is used for these simulations.

5.1 Design #1

In this case, no attempt was made to account for the variations in inertial properties, neither through robustness nor by gain-scheduling. The singular values of the best controller are shown in Fig. 6. The control law resembles a proportional-plus-derivative controller with a steep high frequency roll-off.

In Fig. 7, the response of the linearized closed-loop system to simultaneous 1 degree step commands in the two joint angles is shown at three different values of θ_2. As expected, at the nominal geometry the controller gives a rapid, well-damped response. It should not be surprising that, at the extreme values of θ_2, the stability of the closed-loop system is marginal and the responses very poor. This design thus demonstrates best nominal performance with no gain-scheduling and poor robustness.

5.2 Design #2

In this design, the variations of the inertia matrix were considered as a parametric uncertainty in the robust synthesis formulation. The resulting controller, whose gains are shown in Fig. 6, is therefore not gain-scheduled with respect to θ_2. The inclusion of the parameter robustness requirements in the design problem has the result of reducing the maximum gains of the controller. The notch filter effect at the flexible mode frequencies provides an extra degree of high frequency attenuation.

The time responses, shown in Fig. 7 demonstrate the increased robustness of this design to variations in θ_2. Since our controller is parameter invariant, this is of course at the cost of degraded nominal performance which is considerably slower than in Design #1. This design illustrates the case of robust performance with no gain-scheduling.

5.3 Design #3

By allowing the controller to vary with manipulator geometry, the gain-scheduling technique used in this final design achieves a higher level of performance. We see in Fig. 6 that the controller gains evolve to compensate for the changing inertial properties of the manipulator. This evolution corresponds closely to the variations of the open-loop system shown in Fig. 2.

The linear step responses presented in Fig. 7 are rapid and well-damped at all values of θ_2. The responses at all geometries are much faster than in the previous case. The gain-scheduled controller comes close to reproducing the maximum level of performance demonstrated in Design #1, but for all manipulator configurations.

6 Conclusions

This study has examined the application of three different control law design approaches to a flexible two-link manipulator. The results have shown the importance of successfully balancing the trade-offs between performance, robustness, and gain-scheduling with the least possible level of conservatism. Reducing conservatism means maximizing the use of information available to the controller while limiting robustness to only that which is physically meaningful in the context of an application. The final design technique, an advanced gain-scheduling approach based on parameter-dependent Lyapunov functions, does the best job of exploiting these ideas. Frequency and time domain results for these approaches have been presented to illustrate the advantages and disadvantages of each.

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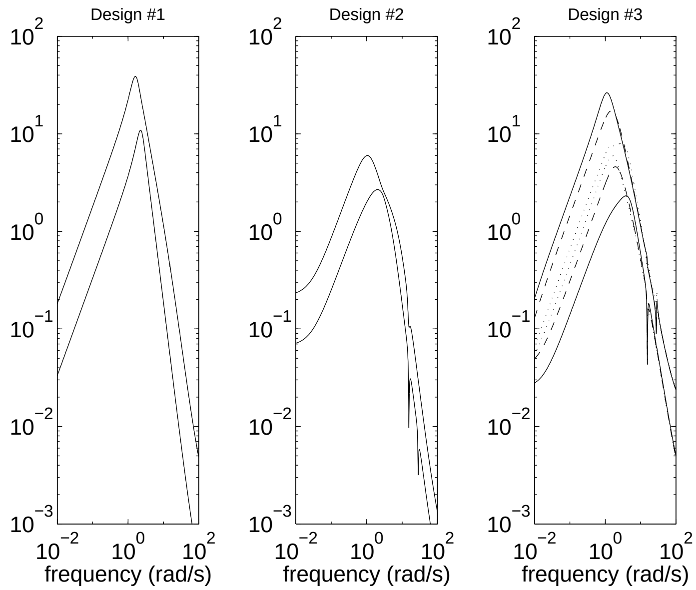

Figure 6: $\sigma(K(s, \bar{\theta}_2))$ for Designs #1-3
$\theta_2 = 0$ (solid), $\pi/2$ (dashed), π (dotted)

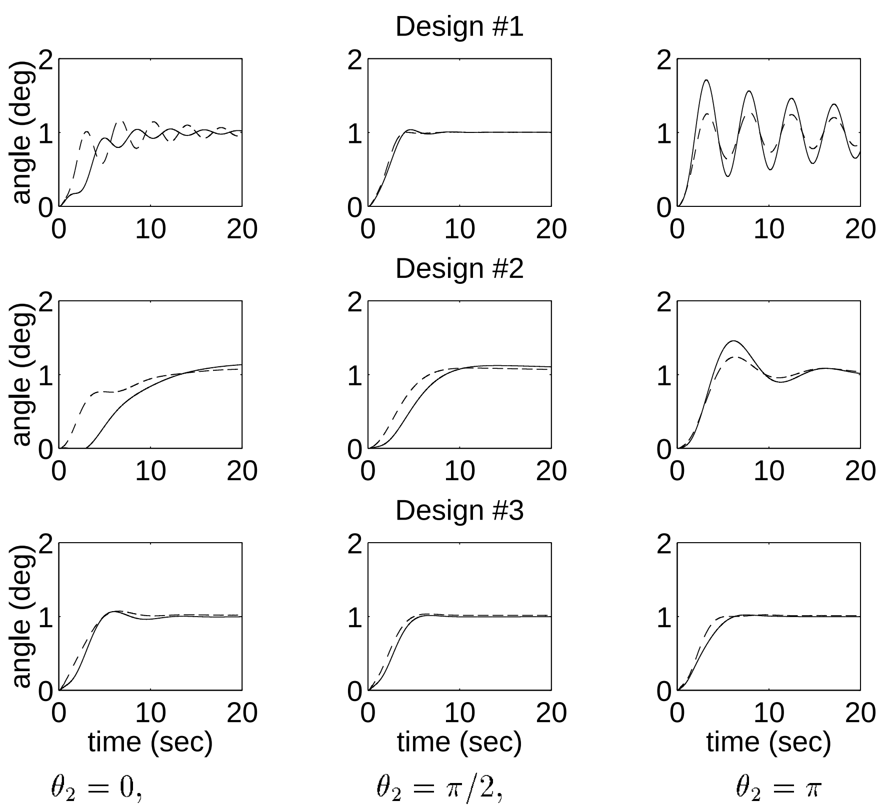

Figure 7: Linear Step Responses for Designs #1-3 at Three Values of θ_2
θ_1 (solid), θ_2 (dashed)