nonsmooth optimization techniques for structured controller design

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outline

- motivation
- nonsmooth optimization
- controller design
- conclusion

context

- challenging practical design problems:
  - reduced- and fixed-order synthesis
  - structured and fixed-architecture synthesis problems
  - general robust control with uncertain and/or nonlinear components
  - simultaneous model/controller design, multimodel control
  - multi-objective frequency- and time-domain designs
  - combinations of the above, etc
- classical design methods (Riccati, LMI) fail on sizeable plants.
- D-K schemes and heuristic methods (homotopy,...) not satisfactory
- solutions based on LMI relaxations often very conservative

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an illustrative example

Boeing 767 at flutter condition

stabilization with static output feedback $u = Ky$:

$$\begin{bmatrix} \dot{x} \\ y \end{bmatrix} = \begin{bmatrix} A & B \\ C & 0 \end{bmatrix} \begin{bmatrix} x \\ u \end{bmatrix}$$

$A = 55 \times 55 \qquad B = 55 \times 2 \qquad C = 2 \times 55 \qquad K = 2 \times 2$

- BMI: $(A + BKC)^T P + P(A + BKC) \prec 0 \quad$ et $\quad P = P^T \succ 0$
  $\Rightarrow$ 1544 variables most of them (1540) for Lyapunov matrix P
- $\min_K \alpha(A + BKC)$ where $\alpha \triangleq \max_i \text{Re}\, \lambda_i$ is spectral abscissa
  $\Rightarrow$ 4 variables : controller parameters

$\Rightarrow$ recast controller design as a nonsmooth program

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controller design as nonsmooth (minimax) optimization

- stabilization

$$\min_K \ \max_i \ \mathrm{Re}\, \lambda_i(A + BKC)$$

- H_∞ synthesis

$$\min_K \ \max_{\omega \in [0,+\infty]} \ \max_i \ \sigma_i\big(T_{w\to z}(K, j\omega)\big)$$

- time-domain design

$$\min_K \ \max_{t \in [0,+\infty]} \ \max\left\{\big(z(K,t) - z_{max}(t)\big)^+, \big(z_{min}(t) - z(K,t)\big)^+\right\}$$

- generic BMI problem

$$\min_x \ \max_i \ \lambda_i\left(A_0 + \sum_k x_k A_k + \sum_k \sum_l x_k x_l B_{kl}\right)$$

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controller design as nonsmooth optimization

- these optimization problems are nonsmooth and/or nonconvex

- special composite structure

$$\max_{x \in X} \lambda_1 \circ T(K, x)$$

where $\max_{x \in X} \lambda_1$ is convex and $T(K, x)$ differentiable wrt. K, $\forall x$

Clarke subdifferential

Clarke regularity $\Rightarrow$ specialized and efficient techniques can be developed

- **directly** formulated in controller parameter space (no Lyapunov variables)
- **flexible**: to handle controller structural constraints
- **efficient**: to handle large control problems

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subdifferential and subgradients

nonsmooth minimax optimization problems of the form

$$\boxed{\min_{K} \underbrace{\max_{x \in X} f(K, x)}_{\triangleq f_\infty(K)}}$$

X stands for:

- a finite set of indices : $X = \{1, \cdots, q\}$

$$\min_{K} \max_{i} \operatorname{Re} \lambda_i(A + BKC)$$

- a time- or frequency-domain closed interval : $X = [0, +\infty]$, $X = [0, t_{max}]$, $X = [\omega_1, \omega_2]$, $\cdots$

$$\min_{K} \max_{t \in [0, +\infty]} \left(z(K, t) - z_{max}(t)\right)$$

- double max : $\min_K \max_{\omega \in [0,+\infty]} \max_i \sigma_i\left(T_{w \to z}(K, j\omega)\right)$.

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assume (for simplicity) that f is continuous and $K \mapsto f(K,x)$ is $\mathcal{C}^1$ $\forall x \in X$.
then f_∞ is loc. Lipschitz and admits a Clarke subdifferential $\forall K$:

$$\boxed{\partial f_\infty(K) = \operatorname*{co}_{x\in\hat{X}(K)} \nabla_K f(K,x)}$$

where $\hat{X}(K) \triangleq \{x \in X \; : \; f_\infty(K) = f(x,K)\}$ active set.
vectors $\phi \in \partial f_\infty(K)$ are Clarke subgradients of f_∞ at K.

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$$\boxed{\min_{K} \underbrace{\max_{x \in X} f(K,x)}_{\triangleq f_\infty(K)}}$$

if f is continuous and $K \mapsto f(K,x)$ is Clarke regular $\forall x \in X$, then f_∞ est locally Lipschitz and has a Clarke subdifferential $\forall K$:

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the $\phi \in \partial f_\infty(K)$'s are Clarke subgradients of f_∞ at K.

nonsmooth optimality condition

Theorem ([?])

$K^\star$ is a local minimum of $f_\infty \Longrightarrow 0 \in \partial f_\infty(K^\star)$

- define optimality function θ to check whether $0 \in \partial f_\infty(K)$ satisfied at K.
- if not, find a descent step $H(K)$ for $f_\infty(K)$, i.e. $\exists t_K > 0$ such that, for all $t \in (0, t_K]$

$$f_\infty(K + tH(K)) < f_\infty(K)$$

- steepest descent:

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- steepest descent:

$$H(K) \triangleq -\text{arg} \underbrace{\min_H \max_{\phi \in \partial f_\infty(K)} \langle \phi, H \rangle + \tfrac{1}{2} \|H\|^2}_{\triangleq \theta_1(K)}$$

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$\Rightarrow K \mapsto H(K)$ not continuous ! convergence problem.

nonsmooth descent direction

$$H(K) \triangleq \arg\min_{H} \max_{x \in \hat{X}_e(K)} \underbrace{f(K,x) + \langle \nabla_K f(K,x), H \rangle + \tfrac{1}{2}\delta \|H\|^2}_{\text{1st-order quadratic local model of } f(\cdot,x) \text{ at } K} - f_\infty(K)$$

$\delta > 0$, $\hat{X}_e(K) \supset \hat{X}(K)$, finite extension of active set

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Proposition ([Apkarian and Noll, 2006-I])

- $\theta(K) \leq 0$ and $\theta(K) = 0 \iff 0 \in \partial f_\infty(K)$
  $\theta(K)$ criticality measure of K for f_∞
- $\theta(K)$ **termination test and (local) optimality certificate**
- $K \mapsto \theta(K)$ continuous
- $f'_\infty(K; H(K)) \leq \theta(K) - \frac{1}{2\delta}\|H(K)\|^2 < 0$
  $H(K)$ descent direction for f_∞ at K
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nonsmooth optimality condition

- primal QP problem

$$\underbrace{\min_{H} \max_{x \in \hat{X}_e(K)} f(K,x) + \langle \nabla_K f(K,x), H \rangle + \tfrac{1}{2}\delta \|H\|^2 - f_\infty(K)}_{\text{optimality function } \theta(K)}$$

- dual QP problem (equivalent)

$$\theta(K) = - \min_{\substack{\tau_x \geq 0 \\ \sum \tau_x = 1}} \sum_{x \in \hat{X}_e(K)} \tau_x \left(f_\infty(K) - f(K,x) \right) + \frac{1}{2\delta} \left\| \sum_{x \in \hat{X}_e(K)} \tau_x \nabla_K f(K,x) \right\|^2$$

with

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- select primal or dual, the one with smaller dimension

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nonsmooth descent algorithm

Algorithm ([Apkarian and Noll, 2006-I, ?])

set $\beta \in (0,1)$, $\delta > 0$, $\varepsilon_\theta > 0$.

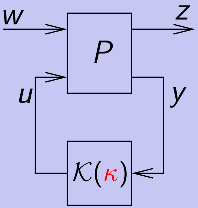
Initialize with controller K_0.

1. set counter $l \leftarrow 0$
2. find active set $\hat{X}(K_l)$.
3. build a finite extension $\hat{X}_e(K_l)$.
4. compute the optimality function value (convex QP) $\theta(K_l)$ and the search direction $H(K_l)$.
5. if $|\theta(K_l)| < \varepsilon_\theta$ (criticality test), stop.
   else compute the step-size t_l such that
   $$f_\infty(K_l + t_l H(K_l)) - f_\infty(K_l) \leq t_l \beta \theta(K_l)$$
6. set $K_{l+1} \leftarrow K_l + t_l H(K_l)$, $l \leftarrow l+1$ and go to step 2.

synthesis formulation

general setup

$$P(s)\left\{\begin{bmatrix}\dot{x}\\ z\\ y\end{bmatrix}=\begin{bmatrix}A & B_1 & B_2\\ C_1 & D_{11} & D_{12}\\ C_2 & D_{21} & 0\end{bmatrix}\begin{bmatrix}x\\ w\\ u\end{bmatrix}\right.$$



- $\mathcal{K}(\kappa)$ defines controller structure
- find κ free parameters to achieve stability, frequency- or time-domain constraints

structured controller design

controller structural constraints

$\mathcal{K}(\kappa)$ with $\kappa \in \mathbb{R}^q$ vector of free controller parameters

structured controller design

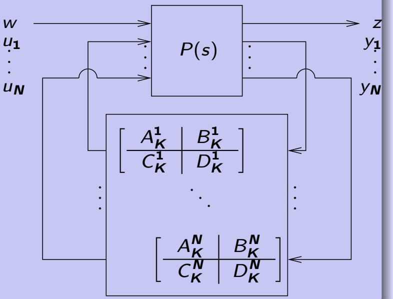
controller structural constraints

$\mathcal{K}(\kappa)$ with $\kappa \in \mathbb{R}^q$ vector of free controller parameters

decentralized control (N sub-compensators)

$$K(\kappa) = \left[\begin{array}{ccc|ccc} A_K^1 & & & B_K^1 & & \\ & \ddots & & & \ddots & \\ & & A_K^N & & & B_K^N \\ \hline C_K^1 & & & D_K^1 & & \\ & \ddots & & & \ddots & \\ & & C_K^N & & & D_K^N \end{array}\right]$$

avec $\kappa = \left[(\text{vec}\, A_K^1)^T \quad \dots \quad (\text{vec}\, D_K^N)^T \right]^T.$



structured controller design

controller structural constraints

$$\mathcal{K}(\kappa) \quad \text{with } \kappa \in \mathbb{R}^q \text{ vector of free controller parameters}$$

fault tolerant control (N sub-systems)

$$K(\kappa) = \left[\begin{array}{ccc|ccc} A_K & & & B_K & & \\ & \ddots & & & \ddots & \\ & & A_K & & & B_K \\ \hline C_K & & & D_K & & \\ & \ddots & & & \ddots & \\ & & C_K & & & D_K \end{array}\right]$$

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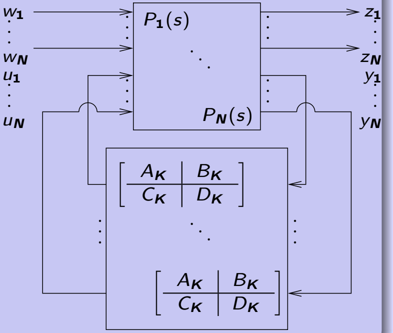

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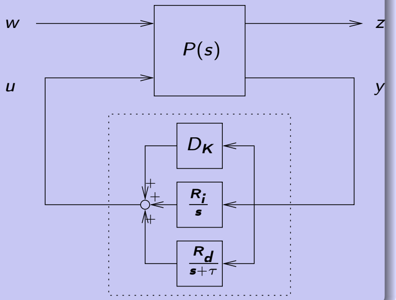
MIMO PID control ($m_2 \times m_2$)

$$K(\kappa) = \left[\begin{array}{cc|c} 0 & 0 & R_i \\ 0 & -\tau I_{m_2} & R_d \\ \hline I_{m_2} & I_{m_2} & D_K \end{array}\right]$$

such that $K(s) = D_K + \frac{R_i}{s} + \frac{R_d}{s+\tau}$.

$\kappa =$

$\begin{bmatrix} \tau & (\text{vec}\, R_i)^T & (\text{vec}\, R_d)^T & (\text{vec}\, D_K)^T \end{bmatrix}^T.$



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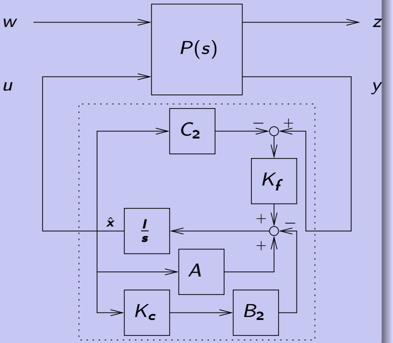
$\mathcal{K}(\kappa)$ with $\kappa \in \mathbb{R}^q$ vector of free controller parameters

observer based control

$$K(\kappa) = \left[\begin{array}{c|c} A - B_2 K_c - K_f C_2 & K_f \\ \hline -K_c & 0 \end{array}\right]$$

K_f: estimation gain
K_c: state-feedback gain

$$\kappa = \left[(\mathrm{vec}\, K_f)^T \quad (\mathrm{vec}\, K_c)^T \right]^T$$



structured controller design

controller structural constraints

$\mathcal{K}(\kappa)$ with $\kappa \in \mathbb{R}^q$ vector of free controller parameters

fractional representations

$$K(s,\kappa) = (N_m s^m + \ldots + N_0)(D_n s^n + \ldots + D_0)^{-1}$$

N_i numerator coefficients
D_j denominator coefficients
$\kappa = \left[\ldots(\mathrm{vec}\, N_i)^T \ldots (\mathrm{vec}\, D_j)^T \ldots\right]^T$

etc

any differentiable possibly nonlinear $\mathcal{K}(\kappa)$

structured controller design

controller structural constraints

$\mathcal{K}(\kappa)$ with $\kappa \in \mathbb{R}^q$ vector of free controller parameters

nonsmooth problem becomes

$$\boxed{\min_{\kappa \in \mathbb{R}^q} \underbrace{\max_{x \in X} f(\mathcal{K}(\kappa), x)}_{f_\infty(\mathcal{K}(\kappa))}}$$

chain rule for subdifferentials with $\mathcal{K} \in \mathcal{C}^1(\mathbb{R}^q)$:

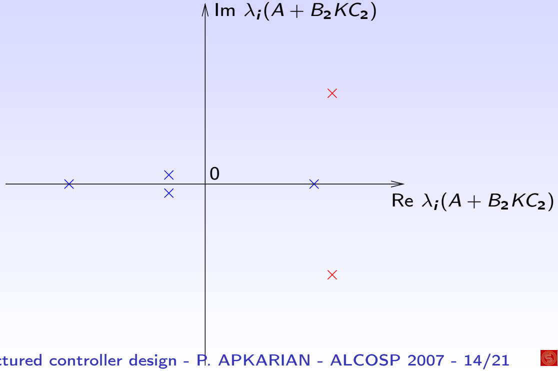
$$\partial(f_\infty \circ \mathcal{K})(\kappa) = \mathcal{K}'(\kappa)^\star \left[\partial f_\infty(\mathcal{K}(\kappa))\right]$$

subgradients $\psi = J_{\mathcal{K}}(\kappa)^T \phi$
where $J_{\mathcal{K}}$ Jacobian matrix of $\mathcal{K}$, and $\phi \in \partial f_\infty(\mathcal{K}(\kappa))$

stabilization

$$\boxed{\min_K \underbrace{\max_i \text{Re } \lambda_i(A + B_2KC_2)}_{f_\infty(K)}}$$

- construction of extended active set $\hat{X}_e(K)$



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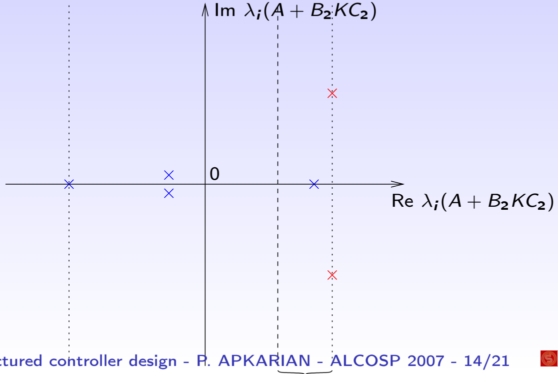

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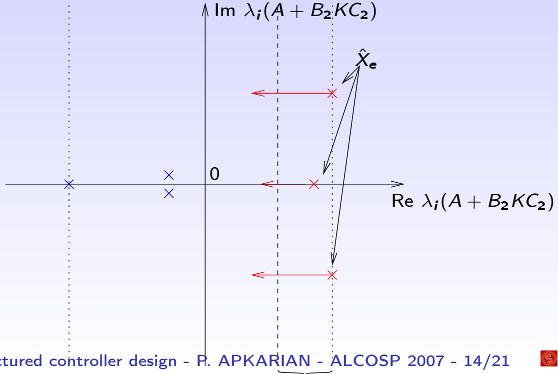

examples from Leibfritz's collection

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AC8 (SOF)	$(9, 1, 5)$	1	< 0.1	9	0.2	$-4.45 \cdot 10^{-1}$	$-5.60 \cdot 10^{-17}$
AC8 ($n_K = 1$)	$(9, 1, 5)$	5	0.2	17	0.4	$-4.45 \cdot 10^{-1}$	$-5.01 \cdot 10^{-27}$
HE2 (SOF)	$(4, 2, 1)$	1	< 0.1	216	2.3	$-2.39 \cdot 10^{-1}$	$-9.77 \cdot 10^{-6}$
HE2 ($n_K = 1$)	$(4, 2, 1)$	1	< 0.1	(28)	0.7	$-2.31 \cdot 10^{-1}$	-1.53
REA2 (SOF)	$(4, 2, 2)$	1	< 0.1	(49)	0.83	-2.46	$-1.1 \cdot 10^{-2}$
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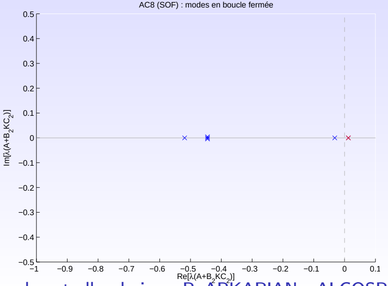

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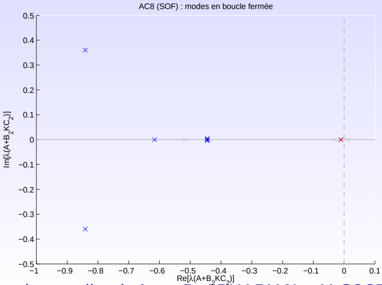

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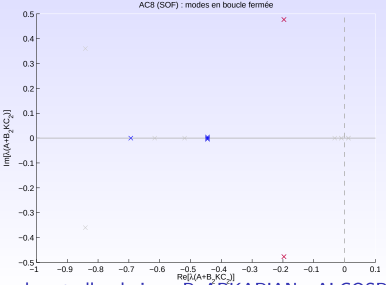

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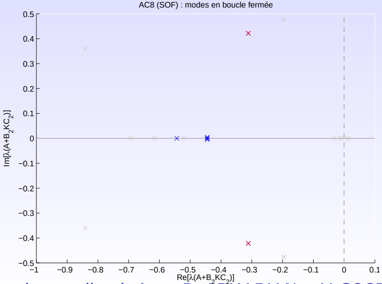

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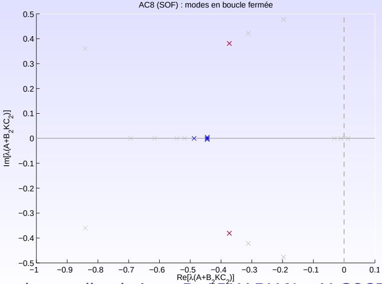

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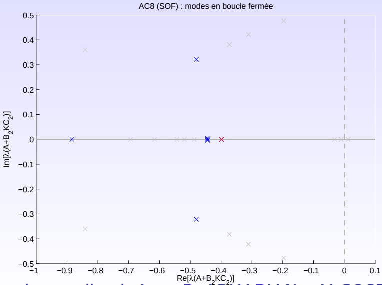

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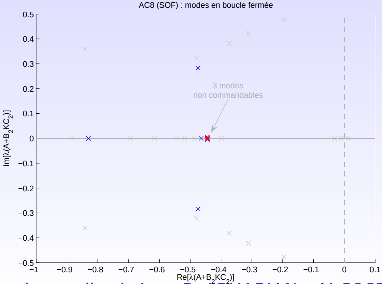
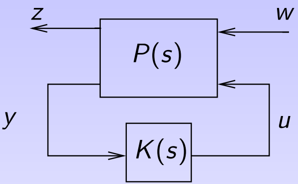


H_∞ synthesis

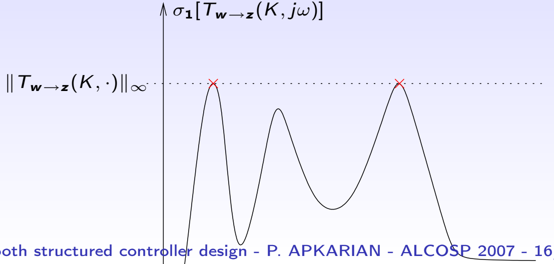
$$\boxed{\min_{K} \underbrace{\max_{\omega\in[0,+\infty]} \max_{i} \sigma_i\left(T_{w\to z}(K,j\omega)\right)}_{=\|T_{w\to z}(K,.)\|_\infty = f_\infty(K)}}$$

$$T_{w\to z}(K(s)) =$$



$$T_{w\to z}(K(s)) := P_{11} + P_{12}K(I - P_{22}K)^{-1}P_{21}$$

- construction of extended active set $\hat{X}_e(K)$



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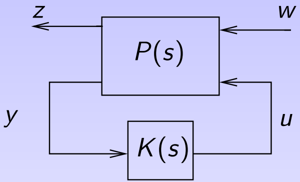

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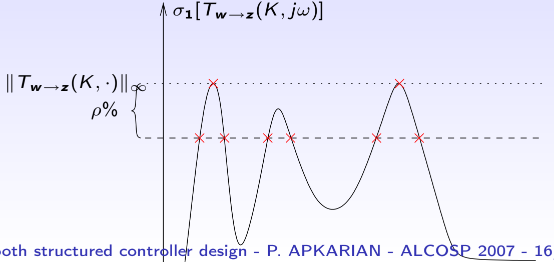

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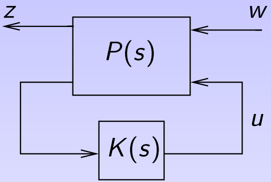

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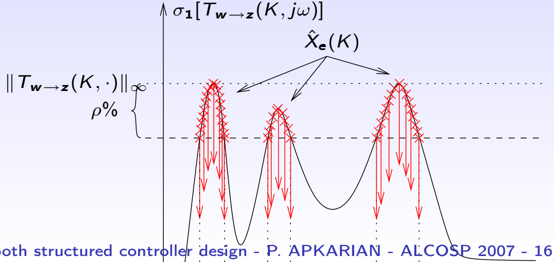
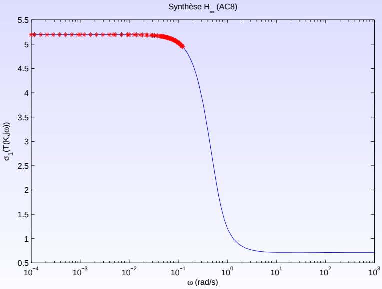


examples from Leibfritz's collection

problem	(n, m, p)	order	iter	cpu (sec.)	nonsmooth H_∞	H_∞ AL	FW	H_∞ full
AC8	$(9, 1, 5)$	0	20	45	2.005	2.02	2.612	1.62
HE1	$(4, 2, 1)$	0	4	7	0.154	0.157	0.215	0.073
REA2	$(4, 2, 2)$	0	31	51	1.192	1.155	1.263	1.141
AC10	$(55, 2, 2)$	0	15	294	13.11	$*$	$*$	3.23
AC10	$(55, 2, 2)$	1	46	408	10.21	$*$	$*$	3.23
BDT2	$(82, 4, 4)$	0	44	1501	0.8364	$*$	$*$	0.2340
HF1	$(130, 1, 2)$	0	11	1112	0.447	$*$	$*$	0.447
CM4	$(240, 1, 2)$	0	2	3052	0.816	$*$	$*$	$*$
CM5	$(480, 1, 2)$	0	2	4785	0.816	$*$	$*$	$*$



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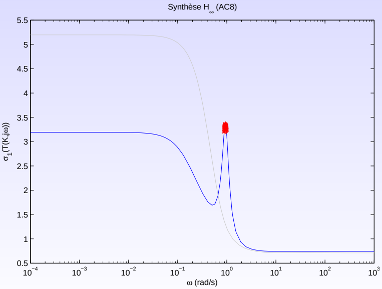

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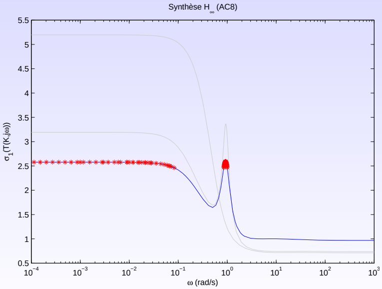

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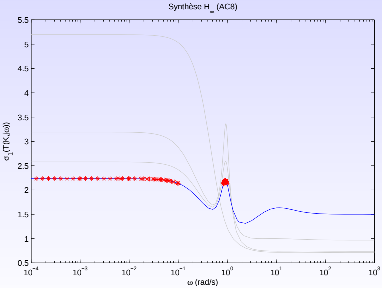

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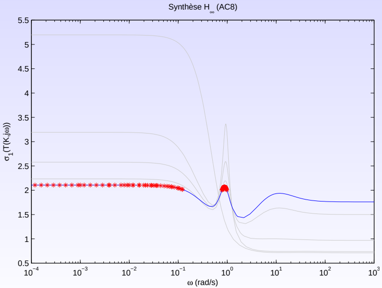

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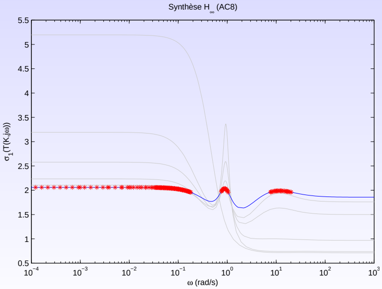
AC8	$(9, 1, 5)$	0	20	45	2.005	2.02	2.612	1.62
HE1	$(4, 2, 1)$	0	4	7	0.154	0.157	0.215	0.073
REA2	$(4, 2, 2)$	0	31	51	1.192	1.155	1.263	1.141
AC10	$(55, 2, 2)$	0	15	294	13.11	$*$	$*$	3.23
AC10	$(55, 2, 2)$	1	46	408	10.21	$*$	$*$	3.23
BDT2	$(82, 4, 4)$	0	44	1501	0.8364	$*$	$*$	0.2340
HF1	$(130, 1, 2)$	0	11	1112	0.447	$*$	$*$	0.447
CM4	$(240, 1, 2)$	0	2	3052	0.816	$*$	$*$	$*$
CM5	$(480, 1, 2)$	0	2	4785	0.816	$*$	$*$	$*$



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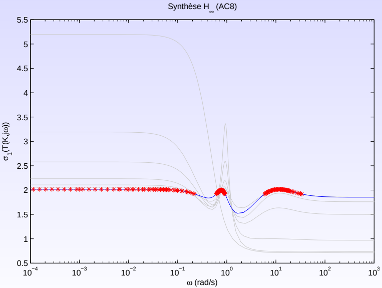

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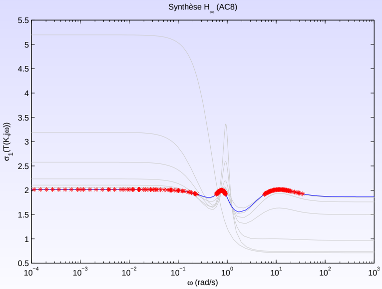

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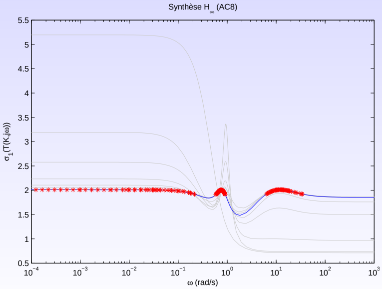

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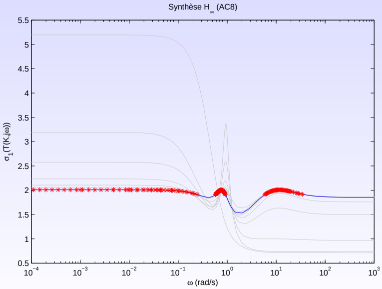

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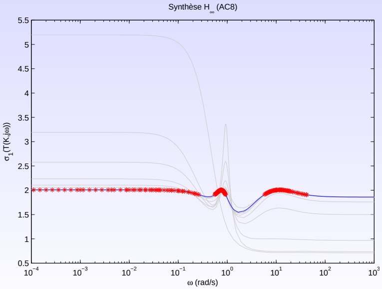


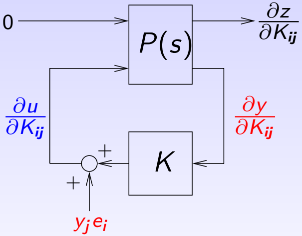
time-domain design

$$\boxed{\min_{K\in} \underbrace{\max_{t\in[0,+\infty]} \left\{ \left(z(K,t) - z_{max}(t)\right)^+, \left(z_{min}(t) - z(K,t)\right)^+ \right\}}_{=f_\infty(K)}}$$

simulated subgradients

$$\phi(K) = \sum_{t_l \in \hat{X}(K)} \pm\tau_l \left[\frac{\partial z}{\partial K_{ij}}(K, t_l) \right]_{i,j}$$

with $\tau_l \geq 0$ and $\sum_{t_l \in \hat{X}(K)} \tau_l = 1$



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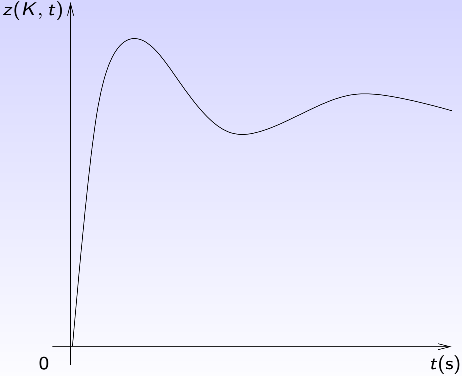

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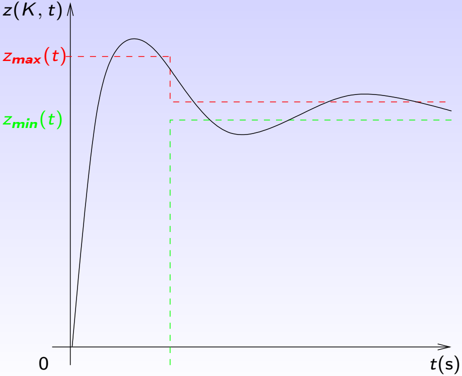

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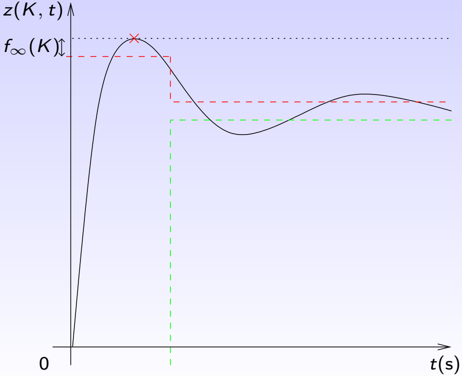

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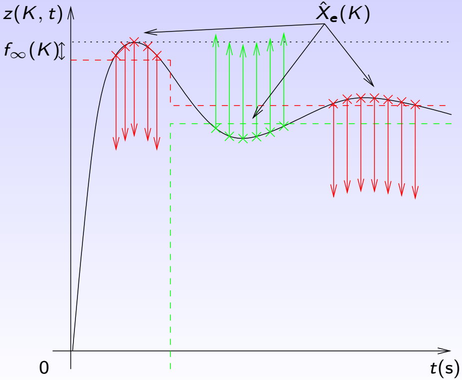

numerical results

2-DOF PID control of a SISO plant

(4th-order)
t_s settling time at $\pm 2\%$
z_{os} overshoot

plant	contr.	t_s (s)	z_{os} (%)
G_1	ZN*	*46.22*	*46.92*
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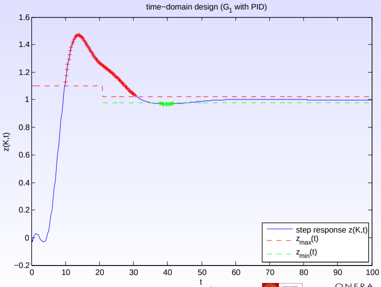
* [Lequin *et al.*, 03]



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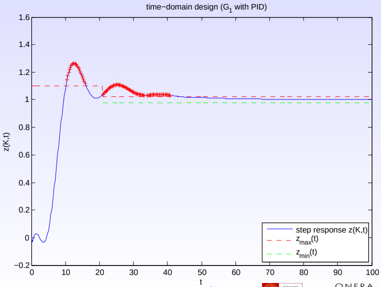

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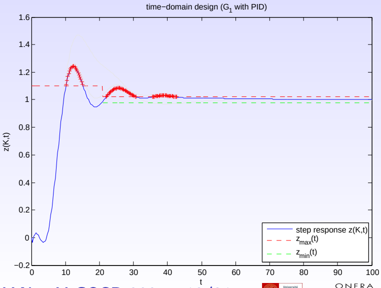

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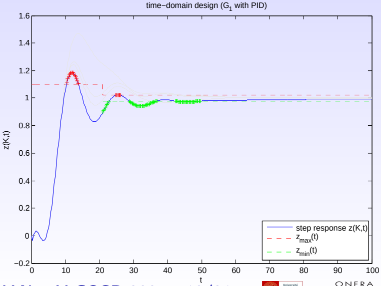

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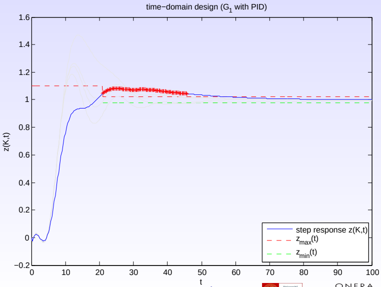

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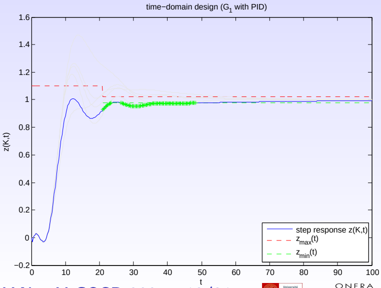

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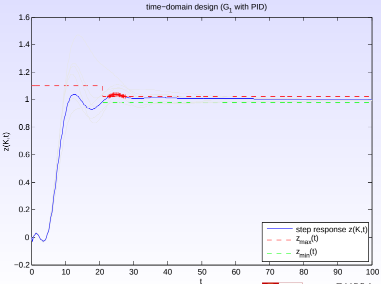

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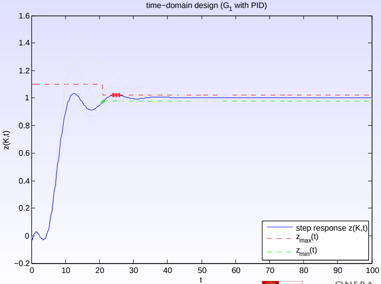

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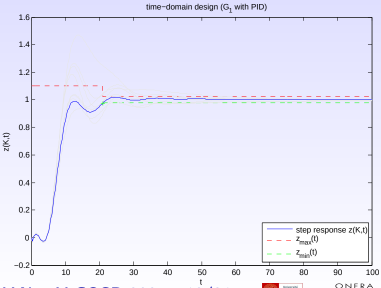

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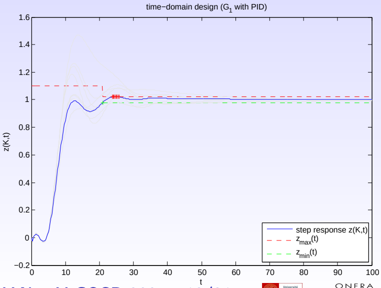

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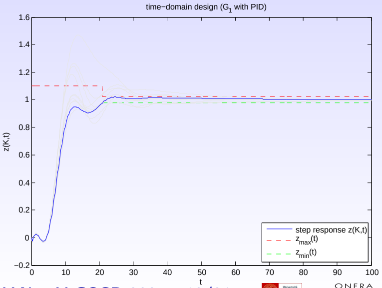

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  - numerically efficient for complex plants
  - flexible for structured practical controllers
  - provide local termination test and optimality certificates
- faster convergence with 2nd order elements:
  - by improving local model θ with BFGS matrix, or
  - by recasting as a smooth local constrained problem and solving with SQP [Bompart *et al.*, 2007]
- Extensions to:
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code and some references

- future release of Matlab
- http://www.cert.fr/dcsd/cdin/apkarian/

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